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PURDUE UNIVERSITY
SCHOOL OF ELECTRICAL ENGINEERING

FIFTH
SEMI-ANNUAL RESEARCH SUMMARY

July through December 1966

SCIENCE TECHNOLOGY AGRICULTURE

Lafayette, Indiana

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RESEARCH AREAS IN THE SCHOOL OF ELECTRICAL ENGINEERING
PURDUE UNIVERSITY, LAFAYETTE, INDIANA

The following descriptionsof research in general areas within the School of Electrical Engineering at Purdue University represent an overview of much of the work being done over a six-month period, or planned in the immediate future. Subsequent reports will, of course, involve much of the same basic programs, with some phases reaching completion while other new projects or areas are added.

Section I

MEDICAL AND LIFE SCIENCES RESEARCH

Biological Systems and Electrophysiology

Research is presently being conducted into the analysis and function of signals being produced by living organisms. Eventually this study will be concerned with the entire central nervous system.

Speech perception, both by humans and by programmed recognition schemes, is being studied. Facilities include acoustic signal recording and analysis apparatus, and multichannel analog-to-digital conversion systems of both medium and high capacity. Some of this research is done in cooperation with the Department of Audiology and Speech Sciences.

Research in the area of bio-engineering is being done in cooperation with the School of Veterinary Science and Medicine at Purdue. One study involves the depressor nerve which originates in the arch of the aorta of swine. This nerve appears to transmit information on blood pressure, rate of change of blood pressure, and chemical characteristics of the blood. Techniques are being developed to ascertain the characteristics of individual receptors from the electrical activity recorded in the gross nerve. A stretch reflex system in cats is also under investigation. This system involves the muscle spindle stretch receptor and a specific reflex pathway in the spinal cord known as the mono synaptic reflex. The present work is concerned with studying the static and dynamic characteristics of the stretch reflex system under closed loop conditions.

As part of a joint effort between Purdue and the Indiana University Medical Center, research is underway on the functional organization of the cerebral cortex. At present, the Direct Cortical Response is under investigation.

Section 2

CONTROL AND INFORMATION SYSTEMS

Research in the field of control and information systems is concerned with the topics of general optimal control theory, design of adaptive and learning control systems, application of optimization techniques to the worst case design of control systems, feedback shift registers and coding theory, pattern recognition, theory of stochastic automata, cybernetics and artificial intelligency, stochastic optimal control, and stability of nonlinear control systems. Purdue's work in the areas of applying learning process and minimax principle to controller design has been continued. Likewise, the study has been extended to quantization problems in digital control systems, satellite tracking systems and "soft-landing" systems in space vehicles.

New work undertaken includes an investigation into the application of feedback and optimization technique to coding problems realizing a transmission rate nearly equal to the classical channel capacity for the first time, as well as threshold logic design and stability of feedback shift registers, and a design of optimal control for solar sail. Also, a major part of the work in the control and information systems area is concerned with the design of "on-line" learning controllers and pattern recognition systems. On-line learning algorithms have been developed for control systems and pattern classification devices. Sequential statistical decision theory has been applied to both feature selection and pattern classification problems.

In the area of cybernetics and artificial intelligence, the mathematical analysis of the brain model proposed, involving neurons plus glia cell media, has been further extended.

A new EAI 680 analog computer with external digital logic capabilities has been installed during this period. It is expected that more simulation works will be carried out in cooperation with the theoretical studies.

Section 3

ELECTRIC POWER SYSTEMS AND ENERGY CONVERSION

Sec. 3.1 ELECTRIC POWER SYSTEMS

The research program in the area of electric power systems is carried out in cooperation with the Purdue Energy Research and Education Center (PEREC). Due to the breadth of the activities which fall within the scope of PEREC, it has been necessary to establish specialized research groups as the need and resources become available. The first of such research groups is the PEREC Computer Applications Group, which engages in research projects of both short-range and long-range interest. Several basic concepts have influenced the development of this Group. Its specific intent is to maintain a vital power engineering education program and to produce graduates who are well prepared to work on computer applications to power system engineering problems. It concentrates on research and stresses its usefulness in supplementing course work. This approach is constructive in giving direction to engineering students interested in power. The research generates student interest for particular courses which are added as the need arises. Research projects are oriented to the needs of industry, and it attracts and utilizes undergraduate and graduate students as well as engineers from the sponsoring companies.

Selection of research projects is based on their suitability for university research, their effectiveness in training students (both graduate and undergraduate), and the competence of faculty members to contribute to the proposed projects. Sponsoring organizations are encouraged to assign an engineer at least half time to the research program. This approach increases the research team's access to relevant data, assists in focusing on the most significant problems of the sponsor, and serves to leave with the sponsor a fund of new knowledge and technical competence which is far more valuable than mere research reports.

General areas of research are more stable than the actual applications which change continuously as work progresses. Topics currently being studied include:

1. Matrix analysis of networks based on exact three-phase representation. This work is being applied to the analysis of unbalance in E.H.V. transmission systems, and also to the study of unbalanced distribution circuits.
2. Electrical load forecasting -- long range for generating planning and short range for operation.
3. Reliability for distribution circuits.
4. Economic scheduling of power with optimum adjustment of voltage levels.
5. Dynamic analysis of power systems during large disturbances.
6. Detailed mathematical models for power plants, particularly boilers.
7. Application of Liapunov's method to determine stability limits.

The Computer Applications Group maintains a comprehensive bibliography of computer applications in power systems. Other reports of general interest are prepared from time to time on topics such as computer science or engineering education.

Sec. 3.2 ELECTRO MECHANICAL ENERGY CONVERSION

Research on electromechanical energy conversion consists of fundamental research in the operation of electrical machines used in aerial and terrestrial transportations either as propulsion or reactive motors or as generators. Also a study of variable speed commutatorless machines for industrial applications is undertaken. In particular, research is being carried on in the use of silicon controlled rectifiers introduced in 1957 in connection with synchronous and induction motors and on the use of semi-conductors in general with rotating machines.

Section 4

ELECTROMAGNETIC FIELDS

The research at Purdue in electromagnetic fields may be divided into four general areas: 1) plasmas, 2) the scattering of electromagnetic waves, 3) wave propagation in a random medium, and 4) the electrohydrodynamic generation of charged particle beams.

The need to maintain communication with space vehicles has led to widespread research on the effects of plasmas on electromagnetic waves and devices. Present work in this area at Purdue consists of studies of the characteristics of antennas operating in a plasma environment.

The vast majority of the work that has been done on plasmas in the past has involved very low intensity perturbations of the plasma, so that the nonlinear plasma equations can be linearized. During the summer of 1966, theoretical work on the nonlinear aspects of plasma behavior was initiated, and plans were made for a considerable expansion of the experimental plasma facility in Electrical Engineering. Some of the experimental work will involve nonlinear phenomena, as does a research project which has been in progress for many months.

A second area of major interest is that of the scattering of electromagnetic waves by material objects -- the exterior boundary-value problem.

This work is of primary importance in the area of the reflectivity characteristics of radar targets. A continuing effort is being devoted to the theoretical study of the reflection of monochromatic electromagnetic waves from perfectly conducting objects having shapes of interest to radar systems designers. Work also is underway on a technique that has been studied relatively little in the past; namely, an attempt is being made to calculate the electromagnetic impulse response of irregular objects. The results are applicable in noise radar systems, and in time dependent scattering problems.

Section 5

ELECTRONIC SYSTEMS RESEARCH LABORATORIES

The purpose of ESRL is to provide an organizational structure for coordinating and managing research progress in the areas and related systems of Communication Sciences, Nonlinear Circuits, and Quantum Electronics.

Sec. 5.1 COMMUNICATIONS SCIENCES

This area is concerned with the general problem of information transmission and processing. Among the various studies presently underway is a major program concerned with adaptive and learning systems. Receivers employing both supervised and unsupervised learning are being synthesized and their performances analyzed. Such systems have a wide range of possible applications, including submarine detection, detection of oil deposits, and pattern recognition.

Research is being conducted in the area of analog communication systems, and in particular with analog demodulation for phase and frequency modulated signals. Current projects include phase lock loop threshold lowering through the use of a module $2n\pi$ linear phase detector, a study of the effects of phase detector characteristics on phase lock loop design parameters, optimum realizable receiver structures for analog modulated signals transmitted through unknown and/or random channels, and a critical evaluation of the available analytical techniques for the analysis of nonlinear systems with stochastic input signals.

Theoretical and experimental studies in the field of radar system design are being conducted, and new methods of signal processing applicable to very wideband systems are under investigation. An experimental random signal radar is undergoing tests at the present time.

Another research area relates to the synthesis of signal waveforms for use in the design of efficient communication systems. One aspect of these studies pertains to signals for use in random-access, discrete-address systems in which a large number of different signals occur in the same frequency band at the same time.

Other studies involve stochastic point processes and their applications to the description of other stochastic processes.

Sec. 5.2 NONLINEAR CIRCUITS AND SYSTEMS

Electronic Circuit theory research can be subdivided into three basic problem areas. The first is the determination of fundamental properties of classes of networks. The second is the synthesis of linear, time-invariant circuits for specific applications, and the third is the study of specific properties of special nonlinear and linear, time-variant circuits. A laboratory program is maintained to complement the theoretical research, especially in the second and third areas.

Problems in the first area include a current study to unify all the approaches and concepts that have proved useful in the analysis and synthesis of linear, time-invariant circuits. The relation between various external port formulations and internal state variable and energy formulations are being studied so that the advantages and disadvantages of each can be readily ascertained. Also in the first area a study of methods of formulating nonlinear circuit equation is continuing. For complex nonlinear circuits with several non-monotonic nonlinear elements, there are many questions to be answered before equations in some form suitable even for numerical solution can be formulated. A more difficult problem is the formulation of nonlinear circuit equations to which Lyapunov theory or Picard iteration, or some other general theory of differential systems applies.

In the second area various classical synthesis problems that have been partially solved in the context of passive, reciprocal networks are being reconsidered for active, nonreciprocal networks. With active circuits, sensitivity and stability of various realizations complicate the synthesis problems. These latter problems are often more difficult than the formal synthesis.

In the third area various nonlinear signal processors that are very useful but not well understood are being investigated. Parametric amplifiers, phased locked loops, and other frequency converters, and detectors are being considered. By re-evaluating the theory of these devices starting from first principles and using such mathematical tools as nonlinear mechanics and functional analysis, a better understanding of nonlinear signal processors, their applications and limitations, should be forthcoming.

Section 6

MATERIALS

It is the purpose of the Electrical Engineering Materials Laboratory to conduct investigations into the electronic properties of materials which are now, or may eventually be, of electrical engineering significance. The efforts of this laboratory, in the past, have spanned the areas of magnetics (primarily thin film magnetics), semi-conductors (germanium, silicon, gallium arsenide), superconductivity, solid state energy conversion (thermoelectrics, photovoltaic, and thermophotovoltaic), and electrochemistry.

Much of the progress that can be made in the area of solid state devices is closely tied with the capability to fabricate these devices. For this reason, considerable effort is being expended to obtain the necessary equipment for precise device fabrication and evaluation. A complete integrated circuit fabricated facility is presently in the process of being completed. This will complement the already existing facilities for thin-film deposition, metallurgical sample preparation, and chemical analysis.

Projects presently active within the materials laboratory include studies of silicon-germanium heterojunctions, niobium oxide diodes, high intensity photovoltaic cells, thin-film transistors, III-V MOS transistors, nonlinear optics, oscillations in GaAs and InSb, switching characteristics of NiFe tape cores, and coupled thin magnetic films. It is anticipated that as soon as the integrated circuits laboratory is in operation additional projects dealing directly with integrated circuits will be initiated.

Section 7

LABORATORY FOR AGRICULTURAL REMOTE SENSING

An interdisciplinary research program at the Laboratory for Agricultural Remote Sensing has researchers from Engineering and the Life Sciences cooperating to develop techniques to remotely sense agricultural situations and automatically assess their important characteristics.

Man's future material well-being rests upon the conservation, utilization, and development of natural resources, and the improved distribution of the products obtained from them. Opportunities for increasing and sustaining the productivity of natural resources and for facilitating product flows are both identified and measured by accurate, comprehensive, and timely information on resource use, availability, productivity, potentials, and other characteristics. The lack of such information is a major obstacle to the economic development of the underdeveloped regions of the world, and a significant obstacle to the formulation of important policies and programs in the more fully developed regions.

Generally, information on characteristics of natural resources and their productivity is obtained from surveys on the ground. These surveys are costly, and in the more remote and inaccessible regions of the world they are difficult to make. During the past few years many scientists have been developing,and to some extent applying, remote sensing techniques for acquiring data from aircraft. More recently, this scientific interest has evolved into consideration of acquiring data from earth-orbiting spacecraft.

Although the range of types of data that might ultimately be acquired by remote sensing techniques from satellite altitudes is not well defined, low altitude experiments tentatively suggest some of the following potential application: (a) identification of the use of land, including the ability to discriminate among crop species,(b) identification of major topographic and soils features, (c) detection of unique soils problems such as salinity, (d) detection of forest fires, (e) determination of the density of plant growth, and (f) early detection of plant diseases. At the present time, basic knowledge is insufficient for these applications to yield consistent and acceptable information from aircraft, much less from earth-orbiting vehicles. In an effort to develop this knowledge, limited research has been initiated by a number of governmental agencies, industries, and research institutions. Two committees in the National Academy of Sciences have stimulated interest in this field of research, and serve as consulting bodies in research planning and execution.

The National Aeronautics and Space Administration is intensely interested in the use of remote sensors from space platforms. These interests embrace not only earth-oriented applications, but also the use of remote sensors to learn more about the physical and biological environments of other planets. Because of these interests, the National Aeronautics and Space Administration is funding remote sensing research conducted by mutually interested governmental agencies, industries, and research institutions.

Prime objectives of the program at Purdue include studies directed towards

1) a better understanding of plant physics, including determination of the changing characteristics of light energy after it enters a plant,
2) a better understanding of the emission and reflectance properties of many biological and physical materials through spectrophotometric analyses in the laboratory, on the ground, and from low altitudes,
3) identifying the single or combined regions in the electromagnetic spectrum for sensing for particular purposes that will yield unique and consistent signatures,
4) determining the attenuation or other effects on radiation as it is acquired from progressively higher altitudes,
5) designing and building sensors best adapted to obtaining data for particular purposes,
6) designing imagery interpretation, reduction, storage, and retrieval systems to facilitate the rapid dissemination of telemetered data,
7) specifying the minimum accuracy standards and quantity requirements of data for potential applications, defining the range of such applications associated with various levels of sensing and data interpretation and management capabilities, and estimating the potential total and marginal benefits of acquiring various types of data over a continuum ranging from data of some positive value to those with maximum usefulness.

The various research projects are conducted within one or more of the program areas shown in Figure 1.

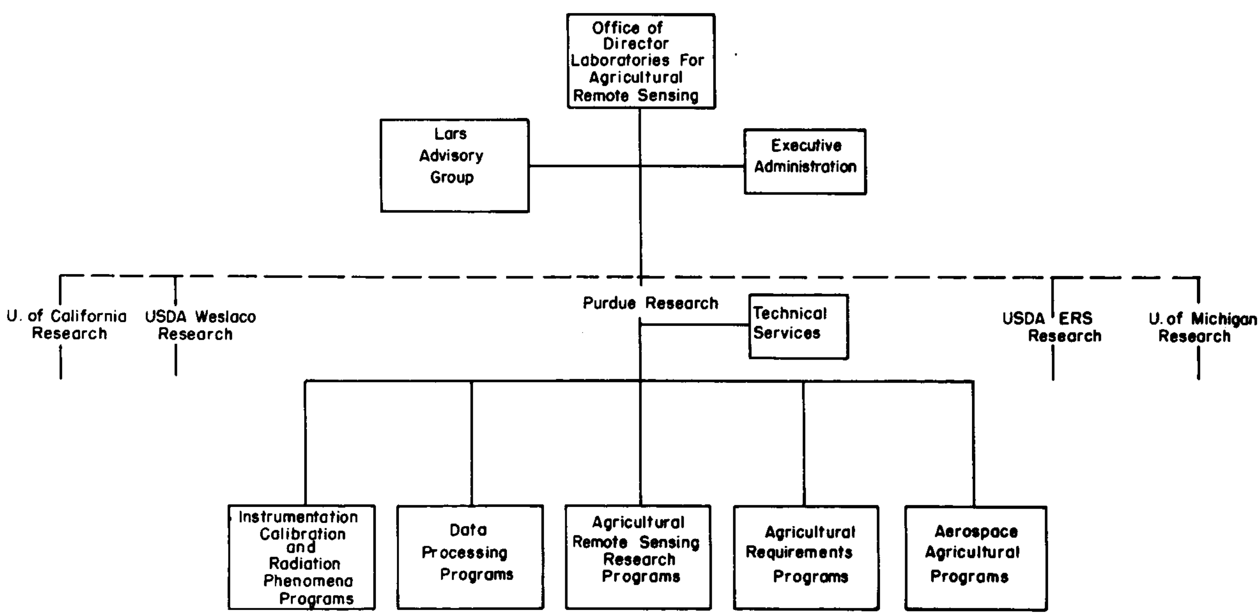

Fig. 1 Program Areas

The Instrumentation Group, under the leadership of Professor Roger Holmes, has undertaken the task of developing instrumentation and related facilities to allow the collection of reliable data for laboratory and field studies. In addition, the group is conducting research to relate observed radiation characteristics of various agricultural features to the physical properties of materials of these features.

The Data Processing Program, under the leadership of Professor David Landgrebe, is organized to carry out work in five categories. These are:

1) Theoretical Pattern Recognition Studies
2) Applied Pattern Recognition
3) Statistical Studies and Modeling
4) Data Handling Research
5) Data Handling Operations

The group receives spectral data gathered in the laboratory (DK-2 Spectrophotometer), the field at 50 ft. altitude (Block interferometer), and from an airborne multichannel scanner. In addition, several types of non-electrically recorded data are also available, and must be processed by the data group. These include photography, agricultural ground truth, and weather data.

Since the formation of the data group in February, 1966, the major effort has been devoted to the establishment of basic data handling and analysis procedures necessary for the reduction of this large quantity and wide variety of data. More pointed and detailed studies have only recently begun.

Dr. Roger Hoffer, Department of Botany and Plant Pathology, directs the Agricultural Remote Sensing Research Programs. This area currently is attempting to determine relationships between observed radiation characteristics and the microphysical and macrophysical properties of radiating sources.

Agricultural Requirements Programs are to establish applications for the sensing techniques: sensing, data interpretation, and management requirements, and estimates of the potential benefits to be expected from such applications. Dr. M. Baumgardner, Agronomy Department, directs this program area.

Aerospace Agricultural programs include studies to define agricultural experiments for future aircraft and space test programs, and to conduct operational system studies to determine requirements of typical future systems.

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SECTION I

MEDICAL AND LIFE SCIENCES

Biological Systems and Electrophysiology

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I. SPEECH COMMUNICATIONS

A. IS THE INTELLIGIBILITY OF ALTERNATED SPEECH RELATED TO RATE OR PLACE OF ALTERNATION?*

George W. Hughes
Arthur S. House**
David P. Goldstein**
John A. Rupf

As part of a study of linquistic factors in the perception of time-distorted speech, we have performed some preliminary experiments in which listeners were asked to repeat (i.e., shadow) speech presented alternately to each ear. Huggins (J. Acoust. Soc. Am. 36, 1964, pp. 1055-1064) has suggested that the interfering effects of alternation should be attributed to some characteristic of the speech signal and are not inherent in the auditory system; he found minimal repetition scores when speech was alternated periodically between listeners' ears at a critical rate in cps close to 80 percent of the talker's average syllable rate. Our experiments evaluated subjects' performance when shadowing speech alternated periodically, and when shadowing speech switched synchronously with syllables. Results showed that when alternation was locked to the syllables, errors in shadowing were twice as frequent as when alternation was periodic at Huggins' critical rate. These findings indicate that syllabic structure of the speech signal cannot be time-altered without serious consequences to perception; distortion of other linguistic units may prove to be equally disturbing.

* This research was supported in part by the Air Force Cambridge Research Laborabories under Contract No. AF 19 (628)-5051. Abstract of a paper delivered before the 72nd meeting of the Acoustical Society of America, Los Angeles, Calif.

** Department of Audrology and Speech Sciences, Purdue University.

B. A LONG-DURATION INHIBITORY PROCESS IN THE CEREBRAL CORTEX

S. Ochs**

F. J. Clark*

The cerebral cortex, presumably the "business end" of the brain, is a complex structure of about 2mm thickness which coats the telencephalon or cerebral hemispheres. This 2mm thick structure can be further subdivided into layers, about 12 in all, on the basis of the organization of various cell groups and nerve fiber meshworks.

If the exposed cerebral cortex is electrically stimulated at its surface via a fine wire electrode, an electrical response can be recorded in the near vicinity of the stimulated site. This response, called the Direct Cortical Response (DCR), consists in the main of a negative spike-like deflection of 10 to 20 msec duration. At low stimulus strengths, the DCR is a simple negative (N) wave, but at higher strengths an after positivity and sometimes a long duration negative wave appear. The appearance of these altered or perhaps new responses with increased stimulus strengths suggest the interaction of at least two or more systems.

The obvious questions to ask in this case are:

1. What structures are activated directly by the stimulus?
2. What paths are involved in the transmission of the activity?
3. What structures respond to produce the direct cortical response?

Answers to these questions were the object of an intensive investigation by the authors this last summer. The results of this study are too voluminous

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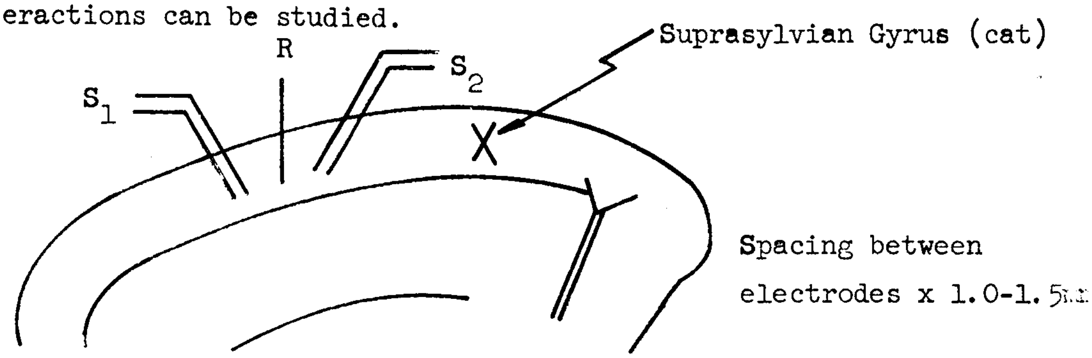
to include in this report and are in preparation for publication in one of the neurophysiological journals. However, one very interesting set of results will be mentioned here.

If two stimulus (S) and one recording (R) electrodes are placed on the cortex as shown, the response to independent stimuli applied to each electrode and their interactions can be studied.



This technique was used in combination with topically applied drugs which inactivate certain structures (GABA 10^{-2} w/v or Tetrodotoxin 10^{-5} or 10^{-6} w/v), plus cuts in the cortex made in such a way as to disrupt transmission in all but a desired layer or layers.

The results of this work indicated that the pathway responsible for information transfer from the stimulated to the recording sites is almost exclusively in the most superficial or molecular layer of the cortex. Also, the structures which produce the DCR are distinctly different than those which respond directly to the stimulus. The response is most likely produced by apical dendrites of deeper cortical pyramidal cells.

One very consistent observation, important from the standpoint of cortical function, was the existence of a long duration inhibitory effect on the N wave DCR. This inhibition could be produced by preceding the test stimulus with a conditionary stimulus to either of the two electrodes. The inhibition was quite pronounced and usually affected only the N wave DCR, leaving the other responses apparently undiminished. This inhibitory effect could be observed after intervals as long as 1.5 seconds. Furthermore, the pathway for the inhibitory effect seemed to be the same as that for the excitatory.

The importance of such phenomenon stems from the apparent fact that cortical activity and its consequences results from exquisitely delicate balances of inhibitory and excitatory activity, both exogeneous and indogeneous relative to an individual neuron. Separating such mutually occurring antagonistic influences is an essential step in our understanding of cortical function. This work will be continued both at the IV Med center and at Purdue as a joint effort between the authors.

N67-33595

II. SIGNAL DETECTION AND TRANSMISSION IN THE NERVOUS SYSTEM

A. UNIT ACTIVITY FROM GROSS NERVES*

E. M. Schmidt

A six channel analog signal delay system has been constructed for sorting of electroneurograms. The system utilizes a Bryant Model C675 magnetic drum. The drum is driven from a variable frequency source allowing a speed range of 600 to 3600 rpm. The combination of head positioning and variable drum speed allows one to delay signals from 0 to 100 msec. Frequency modulation techniques are used on the drum to eliminate the amplitude modulation problem inherent in cheep drums, and to minimize the electronics of the system since the signals to be delayed are recorded on F.M. tape.

The physiological preparation presently under study is the sciatic nerve of the frog. The nerve is excised and placed in a nerve chamber filled with mineral oil to avoid nerve drying. A continuous current is applied to the nerve, which elicits repetitive firing in a number of nerve fibers, similar to the technique of Katz[1]. Nerve firings from a number of sites along the nerve are recorded on magnetic tape for later correlation. Figure 1 shows a typical response at recording sites 2 cm apart. Tape loops are made from these records for analysis by correlation techniques.

The simplest method for correlating the two signals is through delay and addition of the records. A multiple exposure of the sum of the two channels at various delays is shown in Fig. 2. From this record one can establish the delay resulting in maximum signals amplitude, which in turn establishes the conduction velocity and fiber diameter of the nerve fiber in question. The two channels may also be multiplied with various delays between them, resulting in Fig. 3.

*This work has been supported by PHS Grant NB 06156.

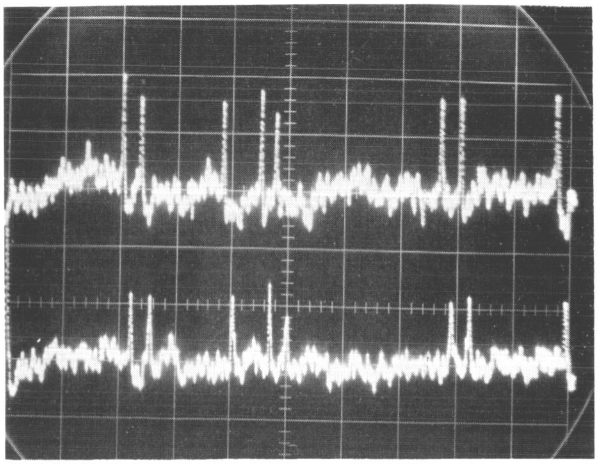


Figure 1. Electrical Activity in Frog Sciatic Nerve Record at Site 2cm A Part.

First Electrode

Second Electrode

sweep speed 2.5 msec/div.

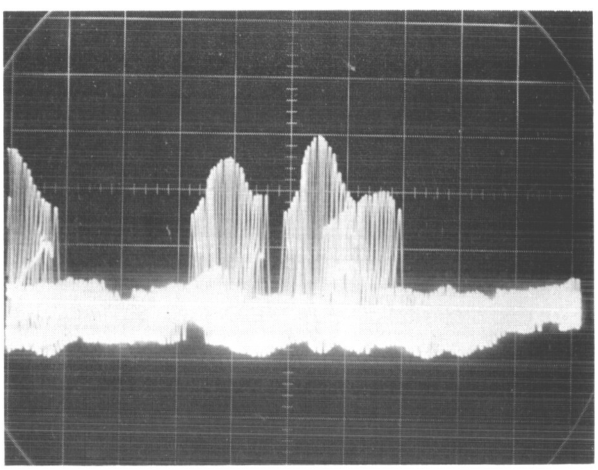


Figure 2. Summation of Center Three Action Potentials of Fig. 1 for Various Delays

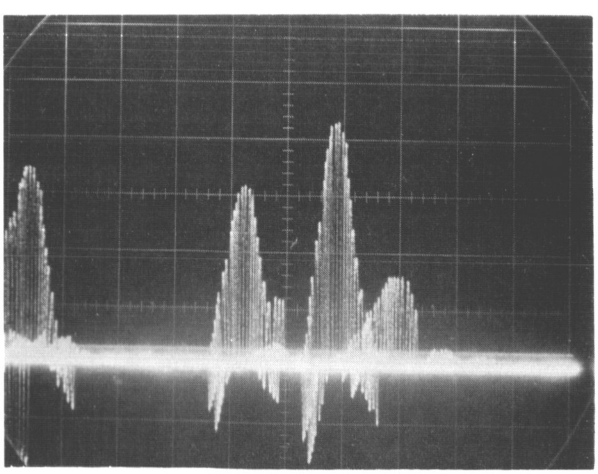


Figure 3. Product of Center Three Action Potentials of Fig. 1 for Various Delays

Again the delay producing maximum amplitude establishes the conduction velocity of the nerve fiber under observation. One notes the increased selectivity of the multiplication process; it may also be possible to use addition in many cases to obtain the desired sorting of nerve fixings.

Using a triangular pulse of width W, one can easily investigate the effects of different electrode arrangements on the peak amplitude of the correlated signals. Figure 4 illustrates the affect of using two, three, and four recording electrodes. The overall spacing of the electrodes has remained constant while additional electrodes are added between the two outside electrodes. It is interesting to note that the normalized peak amplitude for the product of 2 and 3 electrodes results in the same curve, while the summation of 3 electrodes produces a broader curve than the 2 electrode system.

Techniques are under investigation that will provide automatic sorting of individual nerve fibers.

REFERENCE

1. Katz, B., Multiple Responses to Constant Current in Frog's Medullated Nerve, J. Physiol. 88, 1936, 239-255.

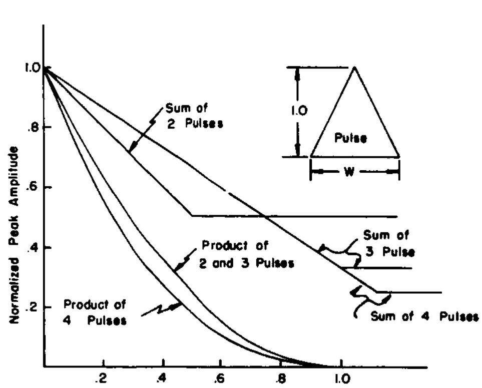


Figure 4. Sums and Products of Ideal Pulses

SECTION 2

CONTROL AND INFORMATION SYSTEMS

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N67-33596

I. OPTIMAL CONTROL THEORY

A. ALGORITHM FOR WORST ERROR IN LINEAR SYSTEMS WITH BOUNDED INPUT AND ITS RATE OF CHANGE*

G. N. Saridis

L. S. Nikora

A computational worst error algorithm for linear systems and quadratic error criteria is considered in this report. The problem is stated in mathematical terms by

Given a plant: $\dot{x} = Ax + br \qquad x(0) = x_o\ ,\ r(0) = 0$ (1)

an error criterion: $E(x(T)) = \frac{1}{2} x^T(T)Qx(T) \qquad Q = Q^T > 0$ (2)

and constraints on the input: $|r| \leq \alpha \qquad |\dot{r}| \leq \beta$ (3)

Find r(t) that maximizes E(x(T)) subject to the constraints.

A flowgraph of the algorithm is presented in Figure 1. It represents a modification of the algorithm proposed in [1], suitable for linear systems, which reduces the computation time by a factor more than 10.

The idea of modification of the algorithm given in [1] stems from the fact that the differential equations of the costate λ are homogeneous and linear. Iterations are performed on the boundary values by setting at the N^{th} iteration

$$\lambda^{N+1}(T) = \frac{\partial E}{\partial x}^N (T) = Qx^N(T) \qquad (4)$$

A backward in time integration of the costate differential equations

$$\dot{\lambda} = - A^T\lambda \qquad (5)$$

is used to obtain the "worst" input at the N^{th} iteration

* This work was supported by NASA, NGR 15-005-021

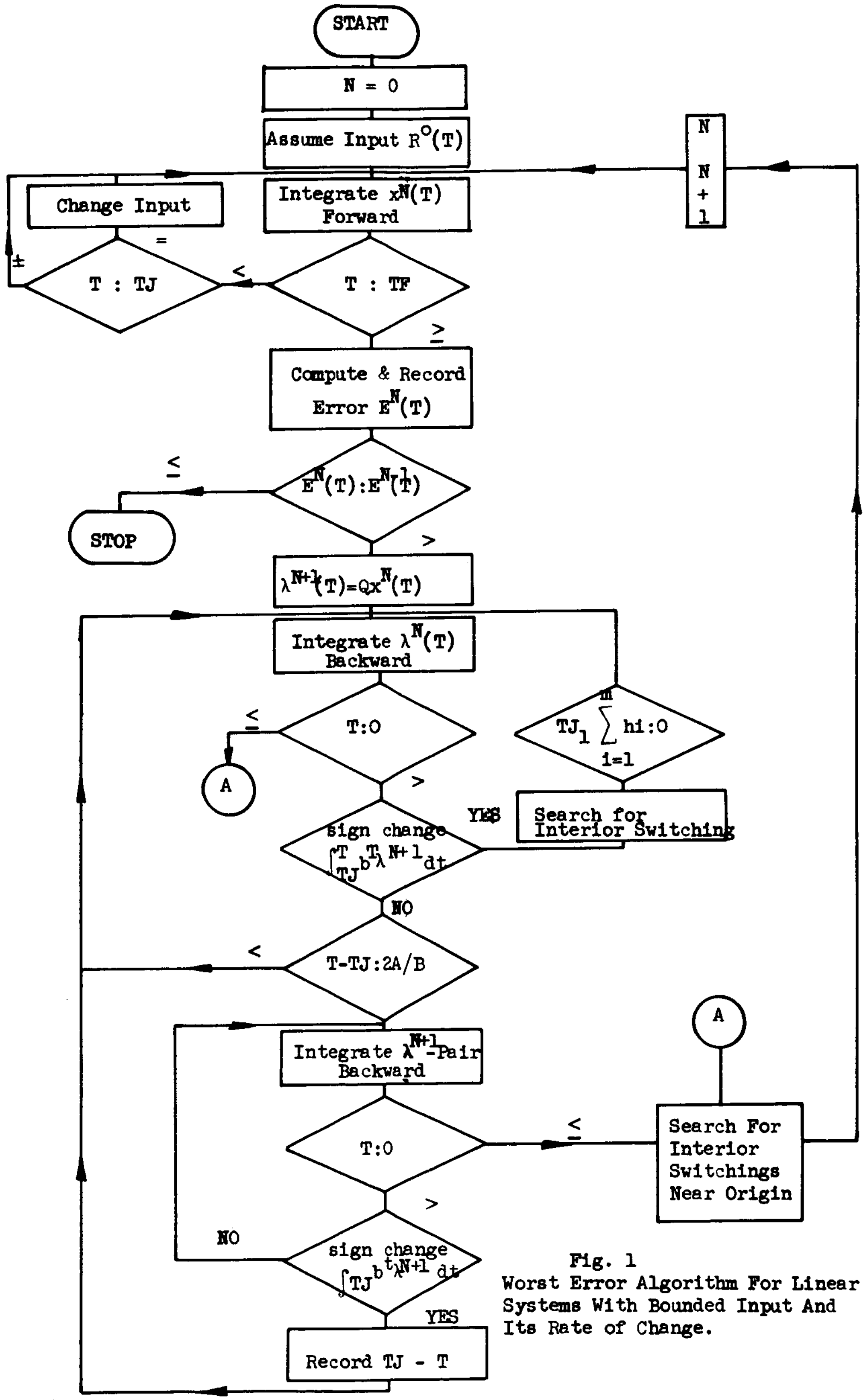

Fig. 1
Worst Error Algorithm For Linear Systems With Bounded Input And Its Rate of Change.

$$\dot{r}(t)^N = \begin{cases} \beta \ \mathrm{sgn} - \int_{t_j^N} b^T\lambda^N(\tau)dt & r^N(t) < \alpha \\ 0 & r^N(t) = \alpha \end{cases} \tag{6}$$

$$\int_{t_j^N} b^T\lambda^N dt \triangleq \int_{t_j^N}^{t} b^T\lambda^N dt \qquad t_j^N < t < t_{j+1}^N$$

where t_j is the j^{th} "switching time" from or onto the boundary[1] satisfying:

$$\int_{t_j^N}^{t_j^N + \sum_{i=1}^{k} h_i^N} b^T\lambda^N(t)\,dt = 0, \ k = 1, 2, \ldots$$

$$\sum_{i=1}^{k} (-1)h_i^N = \begin{cases} 2\alpha/\beta & k \text{ odd} \\ 0 & k \text{ even} \end{cases}, \quad t_{j+1}^N - t_j^N = \sum_{i=1}^{k} h_i^N \tag{7}$$

The equations (1) with (6) are integrated forward in time to yield $x^N(t)$. Thus instability of the costate equations (5) arising from stable plant eigenvalues is avoided. The algorithm converges to a local Maximum of the Error Criterion. A proof of the convergence is presented here.

For sufficiently small $\delta\lambda^N$ the change of the error criterion at the N^{th} iteration is given by

$$\delta E^N(T) = \frac{\partial E^{N^T}}{\partial x^N}(T) \left[\frac{\partial x^N}{\partial \lambda^N}\delta\lambda^N + \delta x^N\right] \tag{8}$$

The first term in the brackets of (8) represents the change of $x^N(T)$ due to the shifting of a fixed number of switching times t_j^N, caused by the change $\delta\lambda^N$[1]. k pairs of switchings are assumed to occur for direct motion from one part of the boundary of $|r| \leq \alpha$ to another, while (m-k) pair are assumed with one interior switching; i.e., $t^N_{2j+1} = t^N_{2j} + 2h^N$. The case of more than one interior switching, which is an immediate generalization of the procedure, is omitted to avoid the resulting complicated algebraic expressions. The proper ordering of the switchings is not important for the proof. Letting $\phi(t)$ be the state transition matrix of (1), one obtains

$$\frac{\partial E^N}{\partial x^N} = Qx^N(T) = \lambda^{N+1}(T) \tag{9}$$

$$x^N(T) = \phi(T)x_o + \int_o^T \phi(T-\tau)br^N(\tau)d\tau$$

$$r^N(t) = \beta\int_o^t \left\{ \sum_{i=0}^{k} \left[u(\rho-t^N_{2i})-u(\rho-t^N_{2i+1})\right]\left[\operatorname{sgn}\int_{t^N_{2i}} -b^T\lambda^N d\eta\right] d\rho \; + \right.$$

$$+\;\beta\int_o^t \sum_{j=k+1}^{m} \left\{ \left[u(\rho-t^N_{2j})-u(\rho-t^N_{2j}-h)\right] \left[\operatorname{sgn}\int_{t^N_{2j}} -b^T\lambda^N d\eta\right]\right.$$

$$\left. + \left[u(\rho-t^N_{2j}-h^N)-u(\rho-t^N_{2j+1})\right]\left[\operatorname{sgn}\int_{t^N_{2j}+h^N} -b^T\lambda^N d\eta\right]\right\} d\rho$$

$$\frac{\partial x^N}{\partial\lambda^N} = \beta\int_o^T \phi(T-t)b\left\{\sum_{i=0}^{k}\left[u(t-t^N_{2i}) - u(t-t^N_{2i+1})\right]\cdot\right.$$

$$\cdot\;\frac{\partial t^N_{2j}}{\partial\lambda^N}\operatorname{sgn}\int_{t^N_{2i}} b^T\lambda^N d\eta + \sum_{j=k+1}^{m}\left[\left[u(t-t^N_{2j})-u(t-t^N_{2j}-h^N)\right]\cdot\right.$$

$$\circ\;\frac{\partial t_j^N}{\partial\lambda^N}\circ\operatorname{sgn}\int_{t^N_{2j}} b^T\lambda^N d\eta + \left[u(t-t^N_{2j}-h^N)-u(t-t^N_{2j+1})\right]\cdot$$

$$\left.\left.\cdot\;\frac{\partial t^N_{2j+1}}{\partial\lambda N}\cdot\operatorname{sgn}\int_{t^N_{2j}} b^T\lambda^N d\eta\right]\right\} dt, \tag{10}$$

$$\delta\lambda^N = \lambda^{N+1}(T) - \lambda^N(T) = Qx^N(T) - \lambda^N(T). \tag{11}$$

where $u(t-t_\alpha)$ is a step applied at t_α .

The second term δx^N, in the brackets of (8), represents the change of $x^N(T)$ due to the possible appearance of new switching pairs at the (N+1)-st iteration which cannot be included in the first term. The case of only one additional switching occurring at the final time interval $[t_{2m+2},T]$ of the process is presented here, to avoid complexity in algebra and demonstrate the behavior of the algorithm for an interval where $T-t_{2m+2} \neq 2\frac{\alpha}{\beta}$. However, the approach is the same for any number of intermediate additional switchings. When such a new switching occurs

$$\delta x^N = \int_{t_{2m+2}^{N+1}}^{T} \phi(T-\tau)br^{N+1}(\tau)d\tau - \int_{t_{2m+2}^{N+1}}^{T} \phi(T-\tau)br^{N}(\tau)d\tau$$

$$= \left[\operatorname{sgn} \int_{t_{2m+2}^{N+1}} b^T\lambda^{N+1}d\eta\right] \int_{t_{2m+2}^{N+1}}^{T} \phi(T-\tau)b\left[2\alpha - \beta(\tau-t_{2m+2}^{N+1})\right]d\tau \qquad (12)$$

where, $r^N(\tau) = -\alpha \operatorname{sgn} \int_{t_{2m+2}}^{N+1} -b^T\lambda^{N+1}d\eta$, $\qquad t_{2m+2}^{N+1} \leq \tau \leq T$

Differentiating (7) with respect to $\lambda^N(T)$ for the proper number of switchings, one obtains

$$\frac{\partial t_{2i}^N}{\partial\lambda^N} = \frac{\partial t_{2i+1}^N}{\partial\lambda^N} = \frac{1}{b^T\lambda^N(t_{2i}^N) - b^T\lambda^N(t_{2i+1}^N)} \int_{t_{2i}^N}^{t_{2i+1}^N} \phi(T-\tau)b\,d\tau \qquad i=1,2\ldots,k$$

$$\frac{\partial t_{2j}^N}{\partial\lambda^N} = \frac{\left[2b^T\lambda^N(t_{2j+1}^N)-b^T\lambda^N(t_{2j}^N+h^N)\right]\int_{t_{2j}^N}^{t_{2j}^N+h^N} \phi(T-\tau)b\, d\tau - b^T\lambda^N(t_{2j}^N+h^N)\int_{t_{2j}^N+h^N}^{t_{2j+1}^N}\phi(T-\tau)b\,d\tau}{f(\lambda^N,\ t_{2j}^N,\ t_{2j+1}^N,\ h^N)}$$

$$\frac{\partial t_{2j+1}^N}{\partial\lambda^N} = \frac{\partial t_{2j}^N}{\partial\lambda^N}+2\frac{\partial h^N}{\partial\lambda^N} = \frac{b^T\lambda^N(t_{2j}^N+h^N)\int_{t_{2j}^N}^{t_{2j}^N+h^N}\phi(T-\tau)b\,d\tau+\left[b^T\lambda^N(t_{2j}^N+h^N)-2b^T\lambda^N(t_{2J}^N)\right]\int_{t_{2j+h^N}^N}^{t_{2j+1}^N}\phi(T-\tau)\ b\,d\tau}{f(\lambda^N,\ t_{2j+1}^N,\ h^N)} \tag{13}$$

where $f(\lambda^N,\ t_{2j}^n,\ t_{2j+1}^N,\ h^N) = 2b^T\lambda^N(t_{2j+1}^N)b^T\lambda^N(t_{2j}^N)-b^T\lambda^N(t_{2j}^N+h^N)\left[b^T\lambda^N(t_{2j+1}^N)+ b^T\lambda^N(t_{2j}^N)\right]$

Substituting (9), (10), (11), (12) and (13) into (8) the following expression is obtained

$$\delta E^N = \beta\left\{\sum_{i=0}^{k}\frac{\left[\int_{t_{2i}^N}^{t_{2i+1}^N} b^T\lambda^{N+1}(\tau)d\tau\right]^2}{\left|b^T\lambda^N(t_{2i+1}^N)-b^T\lambda^N(t_{2i}^N)\right|}+\sum_{j=k+1}^{m}\left[\left|\frac{b^T\lambda^N(b_{2j}^N+h^N)}{f(\lambda^N,t_{2j}^N,t_{2j+1}^N,h^N)}\right|\int_{t_{2n}^N}^{t_{2j+1}^N} b^T\lambda^{N+1}(\tau)d\tau\right]^2\right. +$$

$$+ 2\left|\frac{b^T\lambda^N(t^N_{2j+1})}{f(\lambda^N, t^N_{2j}, t^N_{2j+1}, h^N)}\right| \left[\int_{t^N_{2i}}^{t^N_{2j}+h^N} b^T\lambda^{N+1}(\tau)d\tau\right]^2 + 2\left|\frac{b^T\lambda^N(b^N_{2j}+h^N)}{f(\lambda^N, t^N_{2j}, t^N_{2j+1}, h^N)}\right| \left[\int_{t^N_{2j}+h^N}^{t^N_{2j+1}} b^T\lambda^{N+1}(\tau)d\tau\right]^2 +$$

$$+ \int_{t^{N+1}_{2m+2}}^{T} \left|\int_{t^{N+1}_{2m+2}}^{T} b^T\lambda^{N+1}(\tau)d\tau\right| dt \Big\} \geq 0 \qquad (14)$$

where the equality sign holds <u>only</u> when $\lambda^N(T) = x^N(T)$, $t^N_{2i} = t^{N+1}_{2i}$ and no additional switchings. Therefore, for sufficiently small $\delta\lambda^N$ given by (4) the iteration yields always an improvement of the error criterion. This completes the proof of convergence.

The algorithm was applied to several systems. For the example presented in reference 1, the same results were obtained in 5 iterations or less, but in less than 10 sec of the IBM 7094 computer execution time.

REFERENCE

1. Saridis, G.N., Rekasius, Z.V., "Investigation of Worst Case Errors When Inputs and Their Rate of Change Bounded", IEEE Trans. on Automatic Control, Vol. AC-11, No. 2, April, 1966.

B. AN ERROR ANALYSIS PROBLEM*

V. B. Haas

A. S. Morse

The worst case error analysis problem formulated by Howard and Rekasius is investigated in this project. This is the problem which results when a specific feedback controller is to evaluated in terms of the worst case value of some performance index. In the work that follows it is presumed that the combination of plant and feedback controller results in a linear system.

Problem Statement

Assume the system in question can be represented by the vector differential equation

$$\dot{x}(t) = A(t)\ x(t) + c(t)\ v(t) \tag{2-1}$$

where $x(t)$ is an n-vector describing the state of the system, $A(t)$ and $c(t)$ are time-varying matrices, and $v(t)$ is a scalar forcing function representing a disturbance acting on the system. It is also assumed that an error criterion or performance index of the form

$$J(v(t)) = \frac{1}{2}\ x'(T)\ Qx(T) \tag{2-2}$$

has been defined. In Equation (2-2), prime denotes transpose, T is a fixed time, and Q is a positive semidefinite matrix.

The problem can now be stated. Given some fixed initial time $t_o < T$ and a fixed initial state $x(t_o)$, find a function $v^*(t)$ among all piecewise continuous $v(t)$ functions which are magnitude constrained by

$$\left.\begin{array}{l}\gamma_1(t) \leq v(t) \leq \gamma_2(t) \\ \gamma_1(t) < \gamma_2(t)\end{array}\right\} \quad t \in [t_o, T] \tag{2-3}$$

*This work was supported by NASA, NGR 15-005-021.

such that $J(v^*(t)) \geq J(v(t))$ for all $v(t)$ satisfying (2-3).

Principle Results

Define the function

$$F(x(t),t\alpha) = \frac{1}{2} \left[|x'(t)P(t)\alpha+\theta(t,\alpha)| + \phi(t,\alpha) \right]^2 \tag{2-4}$$

where α is a constant n-vector, $P(t)$ is an nxn matrix satisfying

$$\dot{P}(t) = A'(t)P(t) \tag{2-5}$$

and

$$P(T) = M^{-1} \tag{2-6}$$

M is a nonsingular matrix which diagonalizes Q:

$$MQM' = \left(\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array} \right) \tag{2-7}$$

I_r is an rxr identity matrix, r being the rank of Q. The scalars $\phi(t,\alpha)$ and $\theta(t,\alpha)$ satisfy the differential equations

$$\left.\begin{aligned} \dot{\phi}(t,\alpha) &= - |c'(t)P(t)\alpha| \left(\frac{\gamma_2(t)-\gamma_1(t)}{2} \right) \\ \dot{\theta}(t,\alpha) &= - (c'(t)P(t)\alpha) \left(\frac{\gamma_2(t)+\gamma_1(t)}{2} \right) \end{aligned}\right\} \tag{2-8}$$

with boundary conditions

$$\left.\begin{aligned} \phi(T,\alpha) &= 0 \\ \theta(T,\alpha) &= 0 \end{aligned}\right\} \tag{2-9}$$

Also define the forcing function

$$V(t,\alpha) = \left[\frac{\gamma_2(t)-\gamma_1(t)}{2}\right]\left[\operatorname{sgn}(c'(t)F_x(x(t),t,\alpha)\right] + \left[\frac{\gamma_2(t)+\gamma_1(t)}{2}\right] \qquad (2\text{-}10)$$

Let the r-1 dimensional subspace $A < E^n$ consist of all α vectors such that

$$\left.\begin{aligned} &\sum_{i=1}^{r} \alpha_i^2 = 1 \\ &\alpha_i = 0 \qquad i = r+1 \quad \text{to } n \end{aligned}\right\} \qquad (2\text{-}11)$$

where α_i is the ith component of the α vector. The principle results can now be stated:

1. If a solution to the problem exists, then there also exists an $\alpha^* \epsilon A$ such that

$$J(V(t,\alpha^*)) = J(V^*(t)) \qquad (2\text{-}12)$$

2. The trajectory x(t) satisfies the necessary conditions of the Maximum Principle if and only if

$$F_\alpha(x(t),t,\tilde{\alpha}) = 0; \qquad F(x(t),t,\tilde{\alpha}) \neq 0; \qquad \tilde{\alpha}\epsilon a \qquad (2\text{-}13)$$

3. The relation

$$\left.\begin{aligned} &F(x(t),t,\tilde{\alpha}) = J(V(t,\tilde{\alpha})) \qquad \text{holds if} \\ &F_\alpha(x(t),t,\tilde{\alpha}) = 0; \quad \tilde{\alpha}\epsilon a; \ F(x(t),t,\tilde{\alpha}) \neq 0 \end{aligned}\right\} \qquad (2\text{-}14)$$

4. The relation

$$J(V(t,\alpha)) \geq F(x(t),t,\alpha) \qquad (2\text{-}15)$$

holds for all $\alpha\epsilon a$.

5. If the inequality

$$F(x(t),t,\bar{\alpha}) \geq F(x(t),t,\alpha); \quad \bar{\alpha}\epsilon a \tag{2-16}$$

holds for all $\alpha\epsilon a$ then

$$F(x(t),t,\bar{\alpha}) = J(V(t),\bar{\alpha})) = J(V^*(t)) \tag{2-17}$$

Discussion of Results

It is well known that the problem described may be approached via the Maximum Principle which leads to a two-point boundary value problem. However, a solution of the Maximum Principle equations may not be unique.[1] If several solutions to these equations do exist, they must all be compared to see which one corresponds to the global maximum. Attempting to find all of these solutions is a difficult, if not impossible, task.

The above results mean that the maximization problem described can be transformed into the problem of maximizing a function of r-1 variables on a compact set. Since the function F is not necessarily unimodal, one cannot rely solely on local techniques to find the maximum. However, since the r-1 variables are restricted to lie in a closed, bounded region (an r-1 dimensional hypercube), the maximization of F is viewed as considerably simpler than finding all solutions to the boundary value problem resulting from the Maximum Principle.

REFERENCE

1. Saridis, G.N., Rekasius, Z.V, "Investigation of Worst Case Errors When Inputs and Their Rate of Change Bounded," IEEE Trans. on Automatic Control, Vol. AC-11, No. 2, April, 1966.

C. ON THE CONTROL OF SYSTEMS WITH INACCESSIBLE STATES*

V. B. Haas

S. Murtuza

One of the principle objections to the synthesis of linear control systems based on optimal control theory is that all the state variables are assumed to be accessible for feed-back. This assumption, however, is not always true in practice. The estimation of the inaccessible states may not be desirable from a technical or economical point of view.

One approach to this problem is to synthesize the controller by finding the optimal feed-back coefficients for the accessible states. This approach can not always be used successfully, because it may not be possible to stabilize the system, let alone optimize its performance, by merely feeding back the accessible states. Moreover, even if the system can be stabilized, the resulting controller will be very conservative.

An alternate approach is to use dynamic equalizers instead of using constant coefficient feed-back. In this case it can be shown that the system is guaranteed to be stable, and in addition the performance of the resulting system may be forced to approach the optimal. The solution to this problem involves finding (mix max) of a ratio of two quadratic forms:

$$\min_{y} \quad \max_{x} \quad \frac{x'R(y)x}{x'Px}$$

where x is an n-vector, y is an m-vector, R is an (nxn) positive definite matrix whose elements are functions of y, and P is a constant positive definite matrix.

*This work was supported by NASA NGR15-005-021.

A computational method has been developed to solve this problem. At present this algorithm is being tested. In future this method of synthesis will be extended for the design of multivariable systems with inaccessible states.

REFERENCE

1. Rekasius, Z.V., Seminar Lecture given at School of Electrical Engineering, Purdue University, in Spring 1965.

D. TIME OPTIMAL FEEDBACK CONTROL OF NONLINEAR SECOND ORDER SYSTEMS*

V. B. Haas

A. Boettiger

For the second order system,

$$\ddot{x} + f(x)\dot{x} + x - \beta x^3 = u(t),$$

where β is a positive constant, the control $u(t)$ satisfies $|u(t)| \leq 1$, and arbitrary initial conditions are given, it is desired to find the following:

a) the time-optimal feedback control law

b) an upper bound on the number of allowable switchings,

c) the time optimal feedback control law if $u(t)$ is not bounded but $|\frac{du}{dt}| \leq 1$, whenever $\frac{du}{dt}$ is defined.

Partial results to questions (a) and (b) have been obtained.

*This work was supported by NASA, NGR15-005-021.

E. MINIMUM SENSITIVITY OPTIMAL CONTROL

Dr. V. Haas

A. Steinberg

Consider the following example.

$$
\left.
\begin{array}{lll}
\overset{\circ}{x}_0 = 100\, x_2^2 + 0.25\, u^4 & x_0(o) = o & \\
\overset{\circ}{x}_1 = (u - x_1)^3 & x_1(o) = o & \\
\overset{\circ}{x}_2 = -3\,(u - x_1)^2\, x_2 & x_2(o) = 1 & \\
\overset{\circ}{x}_3 = x_1^2 & x_3(o) = o & x_3(0.1) = o
\end{array}
\right\} \qquad (1)
$$

Find a control $u^*(t)$ that will transfer the system from its initial state to its final state and that will minimize $x_0(0.1)$.

The result is $u^*(t) = o$ and the minimum value of the performance index is $x_0(0.1) = 10$. A check should be made whether relaxing the end conditions would allow for a suboptimal control that would result in a lower performance index.

It was found that relaxing the end conditions would result in a performance index equal to 2.75, a significant improvement.

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F. STOCHASTIC COOPERATIVE GAMES WITH APPLICATIONS*

R. L. Kashyap

A class of 2 person games is formulated in which the 2 players, who are subject to differential constraint, cooperate with each other in attaining the objective, recognizing each other's limitations. Each player has exact knowledge of his own state, but only a noisy measurement of the state of the other. Both of them seek feedback control laws. It is possible to reformulate this problem so that it reduces to the solution of a pair of control problems with 2 criterion functions. This theory is applied to the problem of transmission of information over a noisy channel using a feedback link which may or may not be noisy.

*Submitted to Joint Automatic Control Conference to be held in Philadelphia, June, 1967.

G. QUANTIZATION ERROR IN THE DESIGN OF DIGITAL CONTROL SYSTEMS*

A. J. Koivuniemi

The evolution of the error caused by the amplitude quantization in digital control systems can be studied by introducing bounded disturbances at the points of the quantization. This paper introduces the problem of the quantization in the light of the discrete-time optimum control theory. The determination of the worst effect of the quantization error is reduced to solving a common optimum control problem; the 'worst effect' is defined as the maximum of a chosen performance criterion. For a design, a method is proposed for the minimization of the worst effect of the quantization error. Computational techniques are presented. Examples illustrate the presentation.

*Submitted to Joint Automatic Control Conference to be held in Philadelphia, June, 1967.

H. SYSTEM SENSITIVITY AND OPTIMUM CONTROL THEORY*

A. J. Koivuniemi

Summary system sensitivity to bounded external and internal disturbances is investigated. The system sensitivity is defined as the worst effect of the disturbance on a chosen performance criterion. A unified approach to the analysis as well as to the synthesis is proposed. Computational solutions to the problem are presented. The method is applicable to both continuous-time and discrete-time systems.

*This research was supported in part by NASA under Contract No: 15-00-021.

I. AN APPLICATION OF MISHCHENKO'S PURSUIT PROBLEM TO THE SYNTHESIS OF STOCHASTIC OPTIMAL CONTROLLER*

J. Y. S. Luh

G. E. O'Conner, Jr.

Given a linear autonomous differential system

$$dx = [Ax + Bu(t)]dt + C\,dn$$

where x = m-dimensional state vector,

$u(t)$ = k-dimensional control vector, $k \leq m$,

n = h-dimensional sample vector $(h \leq m)$ of a Wiener-Levy process,

A, B, C = constant matrices with appropriate dimensions.

*This research is sponsored by Jet Propulsion Laboratory, Contract No. 950670.

Let the target set be defined as an ellipsoid:

$$e = \{x \mid x' Q x \leq r^2, \quad Q = Q' > 0\}$$

in which ( )' = transpose of ( ), and Q is a real symmetric positive definite matrix. Let

$$\psi_u(\sigma,\overline{x},T) = \text{Prob}\left[x(\tau) \in e \mid x(\sigma) = \overline{x} \notin e\right]$$

for some $\tau \in [\sigma,\tau]$ where T = a given finite time. It is required to find a control input $u(t)$ on $t \in [\sigma,\tau]$ that maximizes $\psi_u(\sigma,\overline{x},T)$.

The control problem is reformulated[1] so that ψ_u satisfies the backward diffusion equation. The results of Mishchenko's pursuit problem[2] are applied to generate an approximation to ψ_u. It is shown that the feasibility of computing ψ_u dictates the selection of a particular shape of ellipsoid for the target e. This choice of e is proven to be natural in the physical sense.

By an application of the maximum principle, a two-point boundary value problem is formulated. Because of the peculiar expression for ψ_u, the computation of the optimal control is difficult. Several computational schemes for this problem are discussed with emphasis on the difficulties.

The generalization of the problem to a time-varying and nonlinear system perturbed by a non-stationary and state-dependent random disturbance is also discussed.

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J. TIME-OPTIMAL CONTROL OF A BOUNDED PHASE-COORDINATE PROCESS - HIGH ORDER SYSTEMS WITH MULTIPLE CONTROL INPUTS*

J. Y. S. Luh

J. S. Shafran

An analysis of the time-optimal control based on the necessary and sufficient conditions was given previously[1]. The effort was applied to the optimal (scalar) control of a second order unstable booster with actuator position and rate limits. It was shown that the optimal controller can be expressed as an explicit time function. A further investigation of the necessary and sufficient conditions has been made during the past six months. The main effort is applied to the optimal (vector) control of a high order booster whose actuators have position and rate saturations. By combining the published results of Pontryagin[2], Russell[3], Schmaedcke[4], and Paiewonsky[5], it is shown that the optimal control (component-wise) can be represented by a multiple of the signum of a modified adjoint solution. Furthermore, each component of the optimal control relates only to the corresponding component of the adjoint solution. It is also shown that the adjoint solution is continuous except possibly at the end of the time interval of interest.

An example, the optimal autopilot design of a second order oscillatory booster with two control inputs is analyzed. With the aid of the results published previously[6], it is possible to obtain analytic expressions for the optimal control inputs.

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*This research is sponsored by Jet Propulsion Laboratory, Contract No. 950670.

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K. ANALYSIS OF A SPECIFIC SOFT-LANDER CONTROL PROBLEM*

J. Y. S. Luh

J. S. Shafran

A study was made on a specific soft-lander control problem[1]. The dynamic system of the vehicle is de-coupled so that the pitch, yaw, and roll movements can be treated separately. The pitch and yaw are controlled by two independent but indentical torque loops, while the roll is controlled by a thrust loop.

In the torque loop, the smallest angular displacement deviation and the smallest settling time are of interest. The saturation of the output current yields a constraint on the control amplitude, and the time delay in the radar-gain feedback branch results in a dual-mode control system. The problem as a

*This research is sponsored by Jet Propulsion Laboratory, Contract No. 950670.

whole can be solved by an application of the optimal control theory. However, the analysis via the classical method indicates that a proper choice of gain-constants in the feedback branches would achieve less than five per cent of the deviation at the time when the radar-gain loop is switched on.

In the thrust loop, an asymptotical stability of the system is desired. Two nonlinear elements are in the loop with the one in the feed forward branch having hysteresis characteristics. The thrust motor in the loop acts as a low-pass filter. It is found that by an introduction of a time delay element in the loop the asymptotical stability of the thrust loop can be achieved.

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L. MANEUVERABLE LANDING IN MARTIAN ATMOSPHERE*

J. Y. S. Luh

M. P. Lukas

The maneuverable entry of the Martian atmosphere[1] employs a vehicle with a lift capability which is used to adjust the entry trajectory. This technique can be used for soft-landings from orbit or from an Earth-to-Mars trajectory. This technique has an advantage over a retro-rocket landing in that it does not require a sensitive radar and retro-fire system. It makes use of the Martian atmosphere for deceleration, and offers more flexibility in the choice of landing sites.

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*This research is sponsored by Jet Propulsion Laboratory, Contract No. 950670.

The control system parameters depend on the characteristics of the Martian atmosphere, which are not known. These parameters depend also on the vehicle velocity and altitude. Therefore the control system requires a learning and adaptive capability for proper performance under changing or unknown atmospheric conditions. Furthermore, criteria and programs are required for the evaluation of the state of the vehicle during entry. Using this information a decision could be made on the assignment of the priority to one of the following objectives:

a) To sufficiently reduce vehicle velocity so that a parachute landing is possible at the touchdown;

b) To insure that vehicle loading and heating constraints are not exceeded during entry;

c) To counteract previous system errors to hit a prescribed landing site, or to land at a new site.

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II. APPLICATIONS OF ADAPTIVE AND OPTIMAL CONTROL THEORY

A. C.I.S.L. SATELLITE TRACKING SYSTEM*

G. N. Saridis

G. Stein

The work carried out last year involving a crude acquisition system for satellite tracking operations has demonstrated the need for combined identification and control techniques in future development of this system, and particularly in the design of a precision tracking system. The concept of combined identification and control is, of course, a significant problem area in its own right, and its relationship to the tracking problem is used primarily as a motivation for research activity in this area.

The general identification-control problem can be formulated in the following manners:

Given: 1) a plant $\mathscr{S}$ involving certain unknowns (parameters of a differential equation model, for example)

2) Statistics for noise disturbances ω, η

Problem: Using measurements z, develop a control strategy to optimize a specified overall performance index. (Fig. 1)

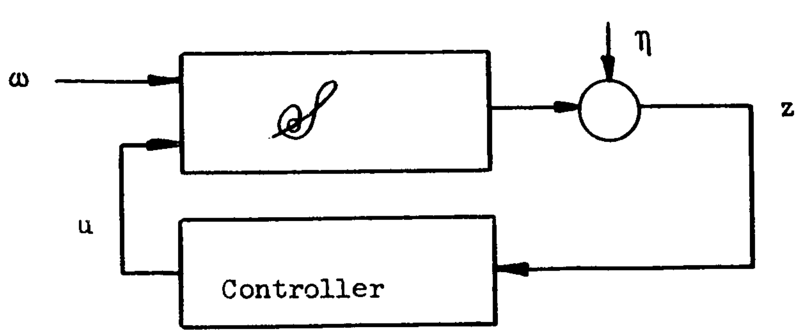

Fig. 1

*This work was supported by NASA, NGR 15-005-021.

The investigation of this problem is being pursued in two principal directions:

1) The separation of identification and control functions

Of concern here is the investigation of existing identification schemes (for example, ref. 2,4,5) to be cascaded with optimal controllers, and the question of overall optimality of such cascaded systems. The results and conjectures of R.C.K. Lee[2] serve as a starting point for the research.

2) The combination of identification and control functions in one control strategy.

This point of view is motivated by the dual-control theory of Feldbaum[3]. The objective here is the development of overall optimal control strategies which simultaneously accomplish the identification and control functions.

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III. STABILITY

A. STATE SPACE PERTURBATION OF THE ZUBOR EQUATION*

M. J. Wozny

Following the direction indicated in the Fourth Semi-Annual Research Summary[1], the present research period has been devoted to the solution of the Zubor Equation via perturbation techniques. The class of nonlinear systems having one singular point and a limit cycle has been considered, and a state space perturbation solution formulated for the problem[2]. This approach is based on, but completely generalizes, an idea originally suggested by Goldwyn and Cox[3].

The features of the present development are outlined next. The arbitrary function in the Zubor Equation (see for example, Reference 1) is evaluated by requiring that each terminated perturbation series be periodic; i.e., for each approximation the state space trajectory be closed. The facing of the periodicity conditions is facilitated by transforming the Zubor Equation into a more suitable set of coordinates. This transformation is determined from the characteristic differential equations associated with the Zubor partial differential equation, and hence, is applicable to nth order systems.

Furthermore, the parameter about which the perturbation solution is expanded need not appear in the system equations, but can be found as a result of a suitable linearization of the nonlinearity.

In contrast, the Goldwyn and Cox paper did not present any systematic formulation for the required transformation, and applied their method only to second order mechanics type systems in which the small parameter is explicitly associated with the damping term.

*This work was supported by NASA, NGR 15-115-021.

The present direction of research is focused on exploiting the full utility of the approach as to the number and types of nonlinearities which can be handled.

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B. CHARACTERIZATION OF GLOBAL STABILITY FOR NONLINEAR SYSTEMS CONTAINING SEVERAL SINGULARITIES*

M. J. Wozny

W. T. Carpenter

Consider a second order nonlinear system having two stable singularities located symmetrically with respect to the origin and a saddle point at the origin. The behavior of such a system is characterized by the separatrix which divides the state plane into two disjoint stable regions, each containing a stable singularity.

This investigation is concerned with developing a method for characterizing the global stability properties of this system. The approach being studied is the following: Let $\underline{x}_s$ represent the singularities of the system, H_s. Let $p(x_1,x_2) = 0$ be the equation of the separatrix which is unknown, a priori, and define two functions $F(x)$ and $M(x)$ which satisfy the conditions

*This work was supported by NASA, NGR 15-115-021.

$F(x) = 1$ for $\underline{x} = \underline{x}_s$

$F(x) = 0$ for $p(x) = 0$

$F(x) > 0$ otherwise

$M(x) = 0$ for $\underline{x} = \underline{x}_s$

$M(x) > 0$ otherwise

Furthermore, define a Liapunov function

$$V = V(F, M)$$

such that

$V = 0$ for $\underline{x} = \underline{x}_s$

$V = \infty$ for $p(x) = 0$

$V > 0$ otherwise

By forcing $\dot{V}$ to be negative definite wrt $\underline{x}_s$ and $\dot{V} \to \infty$ on $p(\underline{x}) = 0$, we find conditions on F and M which specify $p(\underline{x})$.

IV. DIGITAL SYSTEMS AND CODING THEORY

A. LOOP FREE MULTI-LEVEL THRESHOLD LOGIC NETWORK*

W. C. W. Mow

This report deals with the problem of compound threshold element synthesis of an arbitrary Boolean function from the Multi-threshold weight threshold vector (MTWTV). Threshold networks are achieved free from constaints on the thresholds of the MTWTV. The number of single threshold elements required for the synthesis is founded by the number of thresholds of the MTWTV of an arbitrary Booleon function. As Haring and Blair have tabulated, none of the 221 equivalence classes of 4-variable functions under the NPN operation needs more than 5 thresholds in the MTWTV. Thus, using the present technique, all functions of 4-variables can be realized with no more than 5 threshold elements. Symbolically, the k-threshold weight threshold vector is an ordered set w_1, w_2, ..., w_n; T_1, T_2, ..., T_k denoted by $\vec{w}$; $\vec{T}$. It is well known that an arbitrary Boolean function F can be decomposed into k-threshold functions as follows,

$$F = f_1 + f_2 f_3 + \ldots + f_{b-1} f_k \qquad k = \text{odd}$$

$$F = f_1 + f_2 f_3 + \ldots + f_k \qquad k = \text{even} \tag{1}$$

where f_j's are the threshold sub-functions defined by the following weight threshold vectors,

*This work was supported by U.S. Army, Navy and Air Force in the Joint Electronics Service Program, Contract ONR-N00016-66-C0076-A04.

$$f_j: \; w_1, w_2, \ldots, w_n; I_j \qquad j = \text{odd}$$

$$f_j: \; -w_1, -w_2, \ldots, -w_n; -I_j \qquad j = \text{even} \tag{2}$$

The threshold gate, g_{j-1}, realizing the product sub-functions

$$g_{j-1} = f_j \cdot f_{j-1}, \qquad j = \text{odd} \tag{3}$$

is a cascade of 2 single threshold elements as shown in Figure 1(a). Since the sub-functions f_j and f_{j-1} are defined respectively by the weight threshold vector $\vec{W}$; T_j and $-\vec{W}$ $-T_{j-1}$, the weighting of the input variables of g_j and g_{j-1} are $\vec{W}$ and $-\vec{W}$ respectively. Thus, the logic output of g_j is 1 whenever the input excitation exceeds T_j. However, the output of g_{j-1} is equal to $f_j \cdot f_{j-1}$; therefore,

$$g_{j-1} = 1 \text{ iff } T_{j-1} > \vec{W} \cdot \vec{Y} > T_j \qquad j = \text{odd}$$

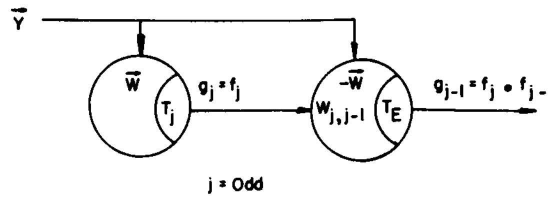
$$= 0 \text{ otherwise} \tag{4}$$

Theorem 1. The product of two sub-functions $f_j \cdot f_{j-1}$, j = odd, is realized by a cascade of two single threshold elements g_j and g_{j-1} where $g_{j-1} = f_j \circ f_{j-1}$ if the weights and thresholds for the threshold gates are determined as follows:

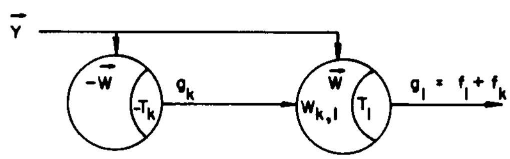
(1) The input wieght vector corresponding to threshold gates g_j and g_{j-1} are respectively $\vec{W}$ and $-\vec{W}$.

(2) The threshold value for gates g_j and g_{j-1} are respectively T_j and $T_E = \frac{1}{2} - \text{MIN } \vec{W} \cdot \vec{Y}$.

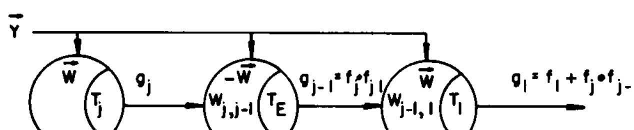
(3) The function input to g_{j-1} from the threshold gate g_j has an input weight of $\omega_{j', j=1} = T_{j-1} + T_E$.



(a) Threshold Network for Product of Subfunctions, $f_j \bullet f_{j-1}$.



(b) Threshold Network for Sum of Two Subfunctions, $f_l + f_k$.



(c) Threshold Network for Sum of Products of Subfunctions, $f_l + f_j \bullet f_{j-1}$

Figure 1. Generation of Threshold Logic Network

Proof: Assume that $T_E = \frac{1}{2} - \text{MIN } \vec{W}\cdot\vec{Y}$ and $\omega_{j,j-1}$ with reference to Figure 1(a). Then

$$g_{j-1} = 1 \text{ iff } g_j \cdot \omega_{j,j-1} - \vec{W}\cdot\vec{Y} > T_E. \tag{5}$$

The excitation will now be divided into 3 regions.

$$\begin{aligned}
&\text{Region I.} && \text{MIN } \vec{W}\cdot\vec{Y} \leq \vec{W}\cdot\vec{Y} < T_j \\
&\text{Region II.} && T_j < \vec{W}\cdot\vec{Y} < T_{j-1} \\
&\text{Region III.} && T_{j-1} < \vec{W}\cdot\vec{Y}
\end{aligned} \tag{6}$$

If $g_{j-1} = 1$ in Region II only, then the theorem is valid. In Region I, $g_j = 0$ and Equation (5) reduces to

$$-\vec{W}\cdot\vec{Y} > \frac{1}{2} - \text{MIN } \vec{W}\cdot\vec{Y} \tag{7}$$

But in Region I,

$$\frac{1}{2} - \text{MIN } \vec{W}\cdot\vec{Y} > -\vec{W}\cdot\vec{Y} \tag{8}$$

is always true, therefore, $g_{j-1} = 0$.

In Region III, $g_j = 1$ and Equation (5) can now be expanded as follows:

$$\omega_{j,j-1} - \vec{W}\cdot\vec{Y} > T_E$$

$$(T_{j-1} + \frac{1}{2} - \text{MIN } \vec{W}\cdot\vec{Y}) - \vec{W}\cdot\vec{Y} > \frac{1}{2} - \text{MIN } \vec{W}\cdot\vec{Y} \tag{9}$$

$$T_{j-1} - \vec{W}\cdot\vec{Y} > 0$$

But in Region III,

$$\vec{W}\cdot\vec{Y} > T_{j-1} \tag{10}$$

implies $g_{j-1} = 0$. Finally in Region II, the threshold gate g_j is again equal to 1, and Equation (5) can be expressed as in Equation (9).
However, in Region II,

$$T_{j-1} - \vec{W}\cdot\vec{Y} > 0 \tag{11}$$

therefore, $g_{j-1} = 1$. Q.E.D.

In the above theorem, g_{j-1} was shown to realize the product of subfunctions $f_j\, f_{j-1}$. Since j is specified odd and arbitrary, the following corollary is true.

Corollary 1. The cascade of 2 single-threshold elements, as shown in Figure 1(a), realizes all products of sub-functions $f_j \cdot f_{j-1}$ for j = odd where $3 \leq j \leq k-1$ for k = even and $3 \leq j \leq k$ for k = odd.

From Equation 1 the given function $F(\vec{Y})$ can now be realized if the product of sub-functions are properly summed.

Theorem 2. The sum of two sub-functions f_1 and f_k, k = even, is realizable by a cascade of two single threshold elements g_1 and g_k where $g_1 = f_1+f_k$, if $g_k = f_k$ is realized by weight threshold vector, $[-\vec{W}; -T_k]$ and $g_1 = f_1 + f_k$ is realized by weight threshold vector, $[\vec{W}, \omega_{k,1}; T_1]$, where $\omega_{k,1} = T_1 - \text{MIN}\ \vec{W}\cdot\vec{Y}$ is the weight of the function input to g_1 from g_k.

Proof: Again, the excitation can be divided into three regions.

$$\text{Region I.} \quad \text{MIN}\ \vec{W}\cdot\vec{Y} \leq \vec{W}\cdot\vec{Y} < T_k$$

$$\text{Region II.} \quad T_k < \vec{W}\cdot\vec{Y} < T_1 \tag{12}$$

$$\text{Region III.} \quad T_1 < \vec{W}\cdot\vec{Y}$$

In Figure 1(b), the network is shown where $g_1 = f_1 + f_k$. Thus

$$g_1 = 1 \quad \text{iff } g_k \cdot \omega_{k,1} + \vec{W}\vec{Y} > T_1 \tag{13}$$

$$= 0 \text{ otherwise}$$

If $g_1 = 1$ in Regions I and II only, then theorem is valid. In Region I, $g_k = 1$ and Equation (13) reduces to

$$\begin{aligned} &\omega_{k,1} + \vec{W}\cdot\vec{Y} > T_1 \\ &[T_1 - \text{MIN}\ \vec{W}\cdot\vec{Y}] + \vec{W}\cdot\vec{Y} > T_1 \\ &\vec{W}\cdot\vec{Y} \geq \text{MIN}\ \vec{W}\cdot\vec{Y} \end{aligned} \tag{14}$$

which is always true in Region I, therefore $g_1 = 1$. In Region III, $g_k = 0$ and Equation (13) is simply

$$\vec{W} \cdot \vec{Y} > T_1 \tag{15}$$

which is exactly the criteria needed for $g_1 = 1$. Finally in Region II, $g_k = 0$, and Equation (15) is not satisfied since $\vec{W}\cdot\vec{Y} < T_1$; therefore, $g_1 = 0$. Q.E.D.

Theorem 3. The sum of products of sub-functions of the form $g_1 = f_1 + f_j \cdot f_{j-1}$ j = odd, is realizable by a cascade of three single threshold elements g_j, g_{j-1} and g_1, if g_{j-1} is realized as shown in Theorem 1 and g_1 is realized by the weight threshold vector $[\vec{W}, \omega_{j-1,1}; T_1]$ where $\omega_{j-1,1}$ is the weight of the function input from g_{j-1} to g_1 and $\omega_{j-1,1} = T_1 - \mathrm{MIN}\ \vec{W}\cdot\vec{Y}|f_i \cdot f_{j-1}=1$. The networks corresponding to Theorem 3 is shown in Figure 1(c). Since all of the products of the sub-functions are disjoint, any arbitrary Boolean function defined by the k-threshold weight threshold vector can be realized by a repeated application of Theorems 2 and 3. The compound threshold networks obtained are shown in Figure 2.

Theorem 4. The Boolean function $F(\vec{Y})$ defined by the MTWTV, $\vec{W};\vec{T}$, can be realized in a compoind threshold network form, if the weights and the threshold values are determined as shown in Figure 2. Theorem 4 follows directly from theorem 2 and 3.

We have utilized the multi-threshold wieght threshold vector in this report to synthesize any arbitrary Boolean function in the compound threshold network form. The structure of the network achieved here is quite different from that which had been presented recently by Haring. Therefore, it gives the logic designer a greater variety of structures to work with. And because of the disjoint nature of the sub-functions and its realization, the threshold network structure presented rendered itself favorably in many areas of logic design.

$T_E = \frac{1}{2} - \text{Min}\ \vec{W}\cdot\vec{Y}$

$W_{j-1,1} = \left[T_1 - \text{Min}\ \vec{W}\cdot\vec{Y} \mid f_j \cdot f_{j-1} = 1\right]$

$W_{j,j-1} = T_{j-1} + T_E$

$3 \le j = \text{Odd} \le k$

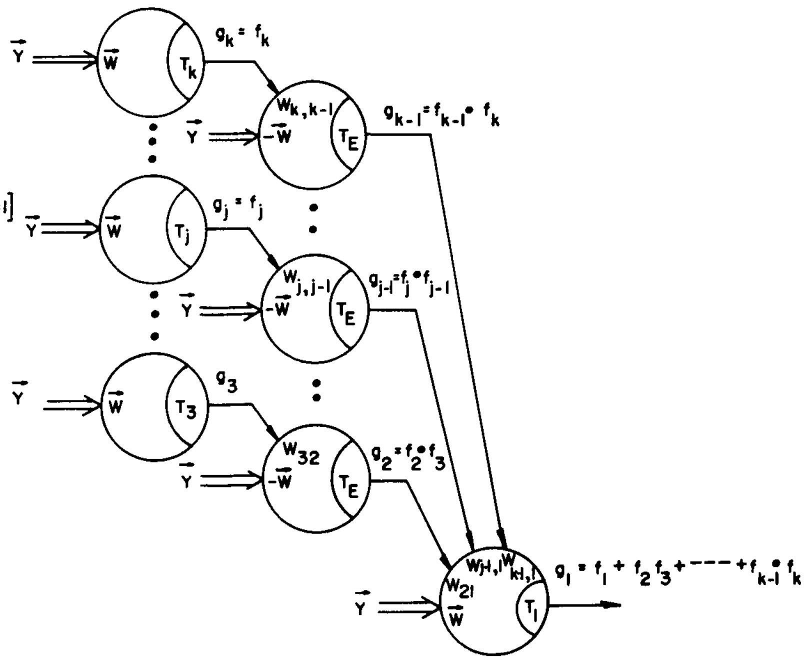


Figure 2 (a) Compound Threshold Network for k=Odd

$T_E = \frac{1}{2} - \text{Min}\ \vec{W}\cdot\vec{Y}$

$W_{j-1,1} = \left[T_1 - \text{Min}\ \vec{W}\cdot\vec{Y} \mid f_j \cdot f_{j-1} = 1\right]$

$3 \le j = \text{odd} \le k+1,\ f_{k+1} = 1$

$W_{j,j-1} = T_{j-1} + T_E$

$3 \le j = \text{odd} \le k-1$

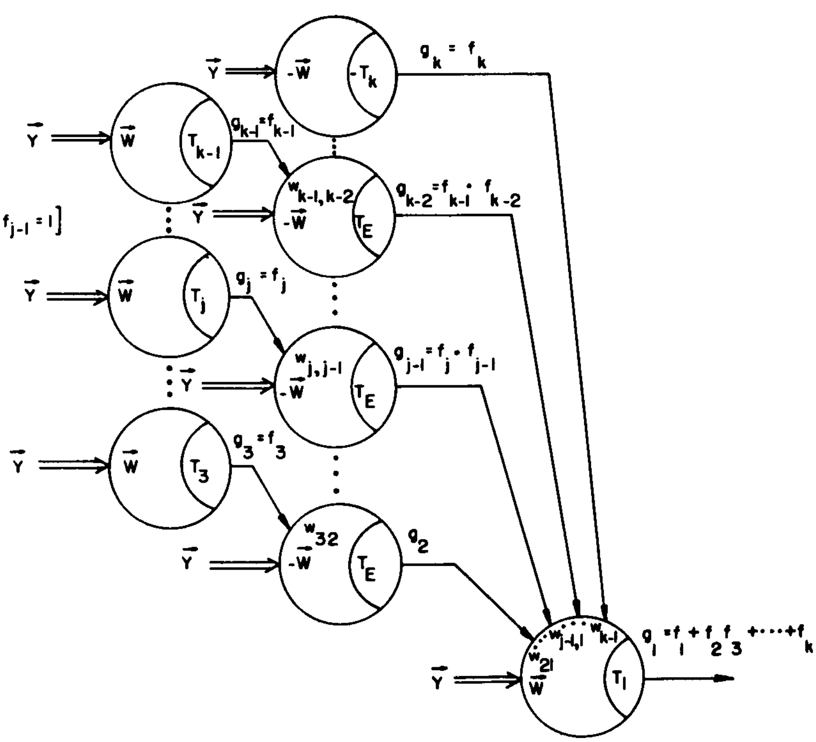


Figure 2(b). Compound Threshold Network For k = Even

It can be seen from Figure 2, that if one assumes the availability of both complemented and uncomplemented input variables, then the equivalent threshold T_E can be reduced to $T_E = \frac{1}{2}$ since the minimum excitation can always be made to be equal to zero. Accordingly, the function input weights $\omega_{j,j-1}$ are proportionally reduced since $\omega_{j,j-1} = T_E + T_{j-1}$.

As can be seen, the proposed synthesis procedures, although not arriving at minimal realizations, do give simple procedures with good results and seem to have many potentially practical applications.

B. SEQUENTIAL CODING SCHEMES FOR AN ADDITIVE NOISE CHANNEL WITH A NOISY FEEDBACK LINK*

R. L. Kashyap

A coding scheme for additive gaussian channel is developed using a noisy feedback link and D-dimensional elementary signals with no band with constraint. This allows error free transmission at a rate $R < R_c$ where R_c is slightly less than the channel capacity C. When there is no noise in the Feedback channel, the coding scheme reduces to a D-dimensional generalization of the coding scheme of Schalkwijk and Kailath. In addition, the expression for the probability of error is determined when T, the time of Transmission rate is finite. Our scheme is also compared with the best codes which use only the forward channel.

*Submitted to the IEEE Transactions in information theory.

C. TOWARD THE UNIFICATION OF AUTOMATA THEORY AND CONTINUOUS SYSTEMS THEORY *

K. S. Fu

P. H. Swain

Systems theorists have occasionally noted similarities between the mathematical forms and operations encountered in dealing with continuous-state systems (eg., differential systems) and discrete-state systems (eg., automata or sequential machines). But little effort has been made to exploit these similarities to the advantage of either type of system. Such exploitation is the goal of the present effort.[1]

Receiving particular attention in this respect are linear, time-invariant differential and sequential systems. Both types of systems may be represented by equations of the form

$$\begin{aligned} x' &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where $x = x(t)$ is the n-dimensional state vector,
$y = y(t)$ is the p-dimensional output vector,
$u = u(t)$ is the r-dimensional input vector
A,B,C,D are nxn, nxr, pxn, and pxr constant matrices, respectively.

For differential systems, $x' = dx/dt$; for sequential systems, $x' = x(t+1)$ (where in the latter case the time variable has been appropriately quantized). Definitions, methods of analysis, and chracteristics common to any system representable in the form (1) are discussed in reference 1.

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* Supported by U.S. Army, Navy and Air Force in the Joint Services Electronics Program, Contract No. ONR, N00016-66-C0076-A04.

The main contribution of this work concerns the transfer function $H(g)$ of a linear system, given by

$$H(g) = \frac{Y(g)}{U(g)}^{*} \tag{2}$$

where $U(g)$ and $Y(g)$ are suitable transforms of the input and output, respectively (such a transform is defined for sequential systems). In particular, a general expression is found relating the constant matrices in (1) to the transfer function. $H(g) = C^T [g^{-1} I - A]^{-1} B - D$. This expression is valid for <u>any</u> type of system expressible in the form (1) if x' and x may be related by $gx' = x$.

The practical significance of this result is that, given any linear time-invariant system, one is able to find the transfer function directly. This representation of the system (in general, a matrix whose elements are ratios of polynomials in the transform variable, g) facilitates certain aspects of analysis, such as determining the zero-state minimal system, and determining uncontrollable and/or unobservable modes of the system. An example in Reference 1 applies this approach to the minimization and realization of a linear sequential system using ternary logic.

The foregoing is an example of how the familiar methods of continuous-state systems theory can be applied to discrete-state problems. Conversely, the algebraic approach of automata theory appears to have promising possibilities for application in the realm of continuous-state systems.

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*That is, the (i,j) element of the pxr matrix $H(g)$ is given by the ratio of the ith component of $Y(g)$ to the jth component of $U(g)$.

D. INVESTIGATION OF THE POTENTIAL FUNCTION METHOD*

K. S. Fu

R. R. Lemke

The process of learning in an unknown environment may be viewed as successive estimation or approximation of unknown parameters of some functional; that is, either parametric or nonparametric estimation. Some of the existing learning techniques are stochastic approximation, reinforcement techniques, potential function method, and Bayesian learning. These can be formulated within the general framework of stochastic approximation. However, the application of these methods as part of a learning system varies considerably due to the different a priori knowledge required. The investigation of the application of the potential function method is of primary concern in this project.

Statement of the Problem

In the potential function method, references 1 and 2, the unknown probability density is assumed to be able to be represented by

$$(1)\quad P(\underline{x}) = \sum_{m=1}^{M} C_m \Phi_m(\underline{x})$$

where the $\Phi_m(\underline{x})$'s, $m = 1,2,\ldots$ are a system of orthonormal functions, and $\underline{x} = (x_1, x_2, \ldots, x_p)$ if it is a joint probability density. The potential function is defined as

$$(2)\quad K(\underline{x},\underline{y}) = \sum_{m=1}^{M} \Phi_m(\underline{x})\, \Phi_m(\underline{y})$$

After the appearance of N random variables from $P(\underline{x})$, $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N$, the estimate of $P(\underline{x})$, $P_N(\underline{x})$, is formed:

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(3) $$P_N(\underline{x}) = \frac{1}{N} \sum_{n=1}^{N} K(\underline{x},\underline{x}_n)$$

It is proved in Equation (2) that as $N \to \infty$, $P_N(\underline{x})$ converges in probability on the average to $P(\underline{x})$.

When considering the application of this method, the following questions arise:

1. What type of orthonormal function should be chosen to give the "best" results?

2. How many terms, M, of $\Phi_m(\underline{x})$ are necessary to guarantee convergence within a given (mean square) error for a fixed number of samples? Tsypkin in reference (3) proposed an investigation of the error introduced by using a finite number of terms, M, of the series in Equation (1).

3. How many samples, N, are necessary to guarantee convergence within a given error for a fixed number of terms, M?

4. Can analytic bounds for the error as a function of N and M be found?

Method of Attack

Substitution of Equation (2) into Equation (3) gives

(4) $$P_N(\underline{x}) = \sum_{m=1}^{N} C_m(N)\ \Phi_m\ (\underline{x})$$ where

(5) $$C_m(N) = \frac{1}{N} \sum_{n=1}^{N} \Phi_m\ (\underline{x}_n)$$

If we let $p(\underline{x})$ be the true probability density to which Equation (1) tends as M , we can define the (mean) square error (reference 4, p. 52.) as

(6) $$\Delta_M = \int_x [p(\underline{x}) - P(\underline{x})]^2\ d\underline{x}$$

This is known to be minimum if the C_m's are the Fourier coefficients, and the value is

$$(7) \quad \Delta_m = \int_X p^2(\underline{x})\, d\underline{x} - \sum_{m=1}^{M} C_m^{\,2}$$

Thus, the best the estimate $P_N(\underline{x})$ can do is Δ_M (based on M terms). Similarly

$$(8) \quad \Delta_N = \int_X [P(x) - P_N(x)]^2\, d\underline{x} = \sum_{m=1}^{M} [C_m - C_m(N)]^2$$

Results

In response to Tsypkin's proposal, the Fourier coefficients based on Legendre, Trigometric, and Hermite orthonormal functions were computed for Gaussian distributions of various means and standard deviations and various uniform distributions. Hence, using Equation (7), the error versus the number of terms, M, can be computed.

The estimated coefficients, $C_m(N)$, which determine $P_N(\underline{x})$, were computed using Legendre, Trigometric, and Hermite orthonormal functions for various values of M and N for the Gaussian and uniform densities. Thus the error can be compared for the various orthonormal functions using Equation (8).

Δ_N is a random variable, and the expected value of it can be related to the known functions. The result is

$$(9) \quad E[\Delta_N] = \frac{1}{N} \sum_{m=1}^{M} \mathrm{VAR}\, [\Phi_m(\underline{x})]$$

This can lead to a theoretical bound for the error as a function of N and M.

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V. LEARNING SYSTEMS AND PATTERN RECOGNITION

A. SIMULATION STUDY OF RELATIONSHIPS BETWEEN SUBGOALS AND PRIMARY GOALS (IP'S) IN A REINFORCEMENT LEARNING CONTROL SYSTEM*

K. S. Fu

L. E. Jones III

A learning control system is expected to learn the solution to an optimal control problem on-line. This learning process is required because of (and proceeds in spite of) incomplete knowledge of the plant and its environment. To design a reinforcement learning control system using the format of Fu and Waltz[1,2,3] the following steps are taken:

1. Discretize time to allow time for decisions and reinforcements.
2. Discretize or quantize the control input so that it can assume only a finite number (K) of choices. The learning controller can then be modeled as a finite automaton[4,5,6].
3. Discretize, quantize, or classify the state variables measured at each sampling instant into control situations.
4. Choose a subgoal to direct the learning process.

The final step has been the subject of an investigation which was first reported in the last research summary[7] and continuing since that time.

The exact relationships between the subgoal and IP were presented in the last summary for the case of a linear, time-invariant plant with a quadratic IP and an unconstrained control. The price of changing these conditions is loss of analytical tractability. Even if we choose to retain the niceties of the linear,

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time invariant plant and quadratic IP, the control must be constrained because of step 2 above. We can now consider that we have a plant described by the following difference equation

$$x(i+1) = \varphi\, x(i) + hu(i) \qquad x(o) = x_o$$

and an IP given by

$$IP = \sum_{i=1}^{N} x^1(i)Qx(i) + \alpha\, u^2(i-1)$$

and $u(i)$ $i=o,1,\ldots,N-1$ is constrained to take only values in the finite set U, where

$$U = \{u_1, \ldots, u_k\}$$

The analytical expression for an exact subgoal for this problem has been sought, but has not been found. The emphasis of the research has shifted toward simulation studies.

A reinforcement learning control system has been simulated on an IBM 7094. During the development of the simulation the following plant was studied.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_2 + 50\, u$$

and

$$u \in U = \{-1, +1\}$$

The plant was discretized in time with a sampling period of 0.25 seconds. The learning time was not a major concern at the outset, though it is important and is taken into account when choosing a reinforcement technique. The primary concern here is to find the best value of an IP which can be learned using a particular subgoal. Therefore, the reinforcement scheme of reference 3 with $b = \frac{1}{2}$, $\gamma = 1.0$, and $C_m = 9$ was used with no attempt to optimize these parameters for fastest learning.

To begin with the sample sets as described in references 1,2,3 were used to classify the state into control situations. This proved to be an excessively time consuming technique compared to a fixed grid quantization. Using sample sets, each classification (after all sets are established) requires computation of the distance of the current state from the centers of each set. Then the minimum distance is found and the state is classified as belonging to this set. The time for a single classification is quite short, but with 50 sets, 500 initial conditions integrated out, and about 15 to 25 sampling periods per initial condition this time became excessive. The sample set technique is not necessarily any less valuable because of this time consumption. It depends entirely upon the speed with which a single classification can be made compared with the available sampling period. For this investigation, though, relative time was of no concern when compared to absolute computer usage time. For this reason a fixed grid was used to classify the state into control situations. And these control situations are squares with predetermined locations rather than circles or elipses which might be located almost anywhere in the space.

It is of interest to note that there has been another case of objection with the sample sets and suggestions for changes in reference 8. However, the suggested changes are not reasonable and make no allowance for the system to have a learning period.

Five initial conditions chosen randomly on the space $|x_1(o)| \leq 50$, $|x_2(o)| \leq 50$ were used between each test trial. The test initial condition was

$$x(o) = \begin{Bmatrix} 50 \\ 0 \end{Bmatrix}$$

and several IP's were calculated as the controller learned to drive the state to the origin. Two stopping conditions were used.

Stopping Condition: N free and determined by

$$x_1^2(N) + x_2^2(N) \leq 25$$

Alternate Stoppeing Condition: N = 21

The alternate stopping condition was used to avoid limit cycles around the terminal manifold. This limit cycling took place for subgoals leading to a switching boundary of $x_1 + x_2 = 0$. The switching boundary divides the state space into two regions. In one region the optimal control is +1 and in the other it is -1. The subgoals reported on here are of the form

$$SG = x'(n)\ G\ x(n)$$

and for this subgoal and the linear plant, the switching boundary is given by

$$h'\ G\ \emptyset\ x = 0$$

The table below summarizes the learned values of these four IP's,

$$IP_1 = N \qquad \text{(which cannot exceed 21)}$$

$$IP_2 = \sum_{i=1}^{N} [x_1^2(i) + x_2^2(i)]$$

$$IP_3 = \sum_{i=1}^{N} [30\ x_1^2(i) + x_2^2(i)]$$

$$IP_4 = \sum_{i=1}^{N} [x_1^2(i) + 5x_2^2\ (i)]$$

where the learning process is directed by one of these three subgoals,

$$SG_1 = 7.67\, x_1^2(i) + x_2^2(i)$$

$$SG_2 = 20\, x_1^2(i) + x_2^2(i)$$

$$SG_3 = 30\, x_1^2(i) + x_2^2(i)$$

Learned Results

Subgoal	Theoretical Switching Boundary	IP_1	IP_2	IP_3	IP_4
SG_1	$x_1 + x_2 = 0$	21	13990	296,100	31,030
SG_2	$1.923\, x_1 + x_2 = 0$	10	13220	235,200	35,490
SG_3	$2.38\, x_1 + x_2 = 0$	10	13220	235,200	35,490

SG_2 and SG_3 seem to yield identical results. Examination of complete data output shows that their learned switching boundaries differ. The choice of test initial condition and sampling period cause the learned test trajectories to be the same because no sample point occurs near enough either switching boundary. SG_1 causes the learned condition of a limit cycle around the terminal manifold so N = 21 cuts the system off. Even with 11 extra sampling periods SG_1 is better than SG_2 or SG_3 with respect to IP_4. The latter two excel in the minimum time case, IP_1.

Additional subgoals which have been tested, with varying results, include the following forms.

$$SG_4 = k_1 \left|x_1(i)\right| + k_2 \left|x_2(i)\right|$$

$$SG_5 = k_1 x_1^4(i) + k_2 x_2^4(i)$$

$$SG_6 = \begin{cases} k_1 x_1^2(i) + k_2 x_2^2(i) & x_1(i), x_2(i) \text{ in region } 1 \\ k_3 x_1^2(i) + k_4 x_2^2(i) & x_1(i), x_2(i) \text{ in region } 2 \end{cases}$$

$$SG_7 = \begin{cases} k_1 x_1^2(i) + k_2 x_2^2(i) & \text{First 100 initial conditions} \\ k_3 x_1^2(i) + k_4 x_2^2(i) & \text{Second 100 initial conditions and cycle back and forth} \end{cases}$$

In addition to the absolute value and quartic form, an attempt is being made to learn a nonlinear switching boundary by using a G matrix (here $g_{11} = k_1$ and k_3, and $g_{22} = k_2$ and k_4, $g_{12} = 0$) which depends upon the state. A time varying switching boundary is required for a time varying plant, so subgoals with time varying G matrices have been considered.

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B. A STUDY OF LEARNING SYSTEMS OPERATING IN UNKNOWN STATIONARY ENVIRONMENTS*

K. S. Fu

Z. J. Nikolic

This study is concerned with the problems of learning which confront the designers of learning control and statistical pattern recognition systems.[1] Learning is considered to be a process of successive approximation or estimation of unknown parameters of a preselected function chosen by the designer to characterize the process under study.

Stochastic approximations are used to describe the problems of learning in unknown time-discrete stationary environments. It is shown that a linear reinforcement algorithm for "learning with a teacher" becomes a stochastic approximation procedure if the expected square error of the estimates of the probabilities of the pattern classes for a given feature vector is desired to converge to zero. Moreover, a more efficient estimator of the unknown probabilities can be obtained on the basis of the same stochastic approximation procedure when the unbiased statistics of the estimated probabilities are available. When

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the number of patterns, signals, or events is countable, the proposed stochastic approximation algorithms can be considered for both "on-line" and "off-line" learning of pattern recognition systems, predictors, and detectors. The same algorithms can also be used for estimation of 1) stationary continuous probability density functions, and 2) the moments of stationary probability distributions defined over the feature space for the pattern classes. It is assumed that the sequence of independent samples from the distribution function under consideration is available.

The successive approximation of unknown parameters in a mixture of a given set of pattern classes is also considered when the correct classifications of the observed samples are not available. The conditions are established under which the parameters of several typical mixtures of the pattern classes can be estimated by using the moments of the mixture. If a system is to be able to learn at all, these conditions must be considered while the designer is selecting the moments of the mixture for estimation.

The procedures for the design of "on-line" learning controllers for unknown time-discrete stationary stochastic plants are described. The plant is, at each instant of control, described by a feature vector which belongs to a feature space preselected by the designer. The control law implemented by the learning controller is a pure random strategy based on subjective probabilities defined over the countable set of available control actions for every observed feature vector. The optimized performance index is the conditional expectation of the instantaneous performance evaluations given the observed feature vector and the applied control action. The sample means of the instantaneous performance evaluations of the available control actions for the give feature vector are used to direct the modification of subjective probabilities of the control actions for the same feature vector in a countable set of feature vectors. Three algorithms for modification of subjective probabilities are described. It is shown that the subjective probability of the optimal action for the given feature vector converges to one with probability one. The algorithms are compared with a linear reinforcement algorithm, and the results of computer simulations are presented.

A unified procedure for the optimal design of both the receptor and the categorizer of a statistical pattern recognition machine is described. It is assumed that the joint probability density function of the available features is known a priori for each pattern class under consideration. If the mutual matching of the receptor and the categorizer is desired, the expected loss in performing classification should be used to describe the quality of the features. The problem of mutual matching of a controller and a plant is also defined.

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C. ON THE FINITE STOPPING RULES AND NONPARAMETRIC TECHNIQUES IN A FEATURE-ORDERED SEQUENTIAL RECOGNITION SYSTEM[1]

K. S. Fu

Y. T. Chien

A class of sequential recognition problems in which the number of feature measurements required to assign class membership to each pattern sample depends on the outcome of the previous measurements is considered. The sequential recognition structure allows the categorizer to observe the feature measurements one at a time, and each time the decision of the categorizer consists of both the choice of closing the sequence of observations and making a terminal decision (namely, assigning class membership to a pattern sample) and the choice of taking another measurement before coming to a terminal decision. The procedure essentially decomposes the single-stage decision process into a multistage decision process, governed by a finite stopping rule which requests no more than the total number of feature measurements available. A merit of this recognition system is that specific decision procedures can be constucted so that, on the average, a substantially fewer number of measurements are required than the equally reliable nonsequential recognition procedures.

The construction of finite stopping rule is first accomplished by considering a modified Wald's sequential probability ratio test in which a pair of time-varying (convergent) stopping boundaries are employed to replace the constant boundaries in the original test. The resulting decision structure is simple and truly sequential, but not necessarily optimal. A second approach to this problem is to consider the dynamic programming method in determining an optimal stopping rule. The decision structure is achieved in the course of solving the the recursive functional equations resulting from the application of Bellman's principle of optimality.

In addition to the finite stopping rules, the problem of designing a non-parametric sequential categorizer is also investigated. No assumption is made in this case as to the form of the underlying probability distributions characterizing the pattern classes. The sequential categorizer computes the likelihood ratios based on the set of sequential ranks rather than on the actual (or assumed) distributions of the measurements. Some analytical and experimental results are obtained to illustrate the feasibility of this approach.

Finally, a procedure for selecting and ordering the successive feature measurements observed by the sequential categorizer is given. The procedure consists of establishing a mutually uncorrelated coordinate system (the generalized Karhunen-Loève system) among all the pattern classes under consideration. The coordinate system is optimal in the sense of minimizing both the mean square error and the entropy function when finite measurements are used to represent the pattern samples.

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D. ADAPTIVE TRAINING SETS FOR LINEAR CLASSIFICATIONS*

K. S. Fu

W. G. Wee

The development of the idea of adaptive training sets deems necessary due to the two extreme cases of procedures generated in the problem of linear classifications; namely, (1) the single-pattern adaptation procedure to the solution of linear classifications. (2) the many-pattern adaptation procedure to the solution of linear classifications. In type (1) single training pattern is used for each correction or adaptation procedure. This is the perception convergence approach to the solution of linear classifications. This type of adaptation mainly consists of the fixed-increment adaptation[1] and the non-fixed increment adaptation[1]. In type (2), all the training samples are kept in the memory in some form. This type of procedure consists primarily of the linear programming approach and the Ho and Kashyap procedure[3] to the solution of linear classifications.

The shortcomings of the type (1) training procedure are that (a) it takes too much corrective or adaptive iterations before a solution is attained if the solution exists; (b) there is no way one can test the existence of such a solution. All these problems can be solved by type (2) training procedure, but the shortcoming of this procedure is that it requires storing the complete training samples. It takes too many storage locations when the sample size is large. It also involves more computations.

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After the above explanation, it is obvious that the idea of adaptation training sets must be developed. With these goals in mind, a much more general training procedure is generated for two classes. Convergence proof shows that such a procedure terminates in finite member of adaptations if the solution exists. Above all such procedure reduces the upper bound of the number of adaptations of type (1) training procedure. As a coincidental by-product, such procedure can be implemented with small additional complexity to the standard type (1) procedure. To test for linear separability, a procedure is developed on the necessary and sufficient conditions for linear separability on each subset of (n+1) training samples for n-dimensional space. Again the computations involved are rather simple.

Generalization of such training procedure is also developed for piecewise linear discriminant functions for multiclass problems. Such a procedure also reduces the upper bound of the number of adaptations of the procedure developed by both Kelser or Nilsson[1], and Duda and Fossum[2]. Again the procedure can be implemented with small additional complexity to the existing system. If one follows the Nilsson[1] approach, linear separability can also be tested in the mid of adaptation.

Computer simulation has been carried out for a two-class problem. A fairly good result has been obtained. Fifteen training samples for each class are generated randomly in a four dimensional Euclidean space. They are linearly separable. With a total of 30 samples, training procedures with different memory capacities are carried out. The result is shown in Figure 1. The total of adaptation before a solution is attained.

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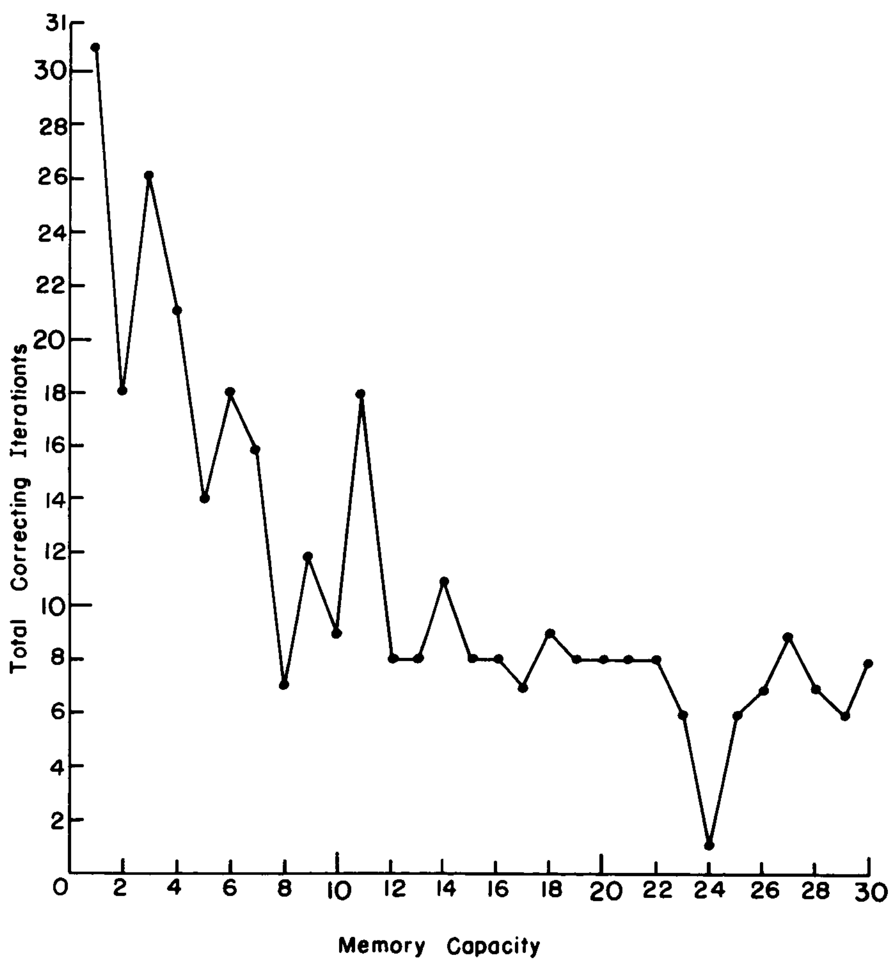

Figure 1.

SECTION 3.1

ELECTRICAL POWER SYSTEMS

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N67-33601

I. SHORT RANGE RESEARCH

A. LOAD FORECASTING

K. N. Stanton

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a. Long Range Forecasting

The exponentially-weighted-time-polynomial-regression has been chosen as the mathematical model to be used for long range forecasting. The data composed of monthly and weekly peak demands or sales is regarded as consisting of trends, weather sensitive variations, and random components, each generated by different phenomena. Separation of the data into its components is continuing, with load forecasting for seasonal weather variations still under investigation. The main regression program is complete, and using seasonally adjusted data, it gives long range forecasts with confidence limits. Work using data from the sponsoring companies is continuing. (See Figure 1).

b. Short Range Forecasting

Forecasting for periods of one to four days is under consideration. Important factors are weather and energy usage patterns. Detailed studies are being delayed to gain maximum advantage from the work on long range forecasting. Also there is a shortage of funds and manpower, and this project was a logical one with which to proceed slowly. A company representative is now needed for this project.

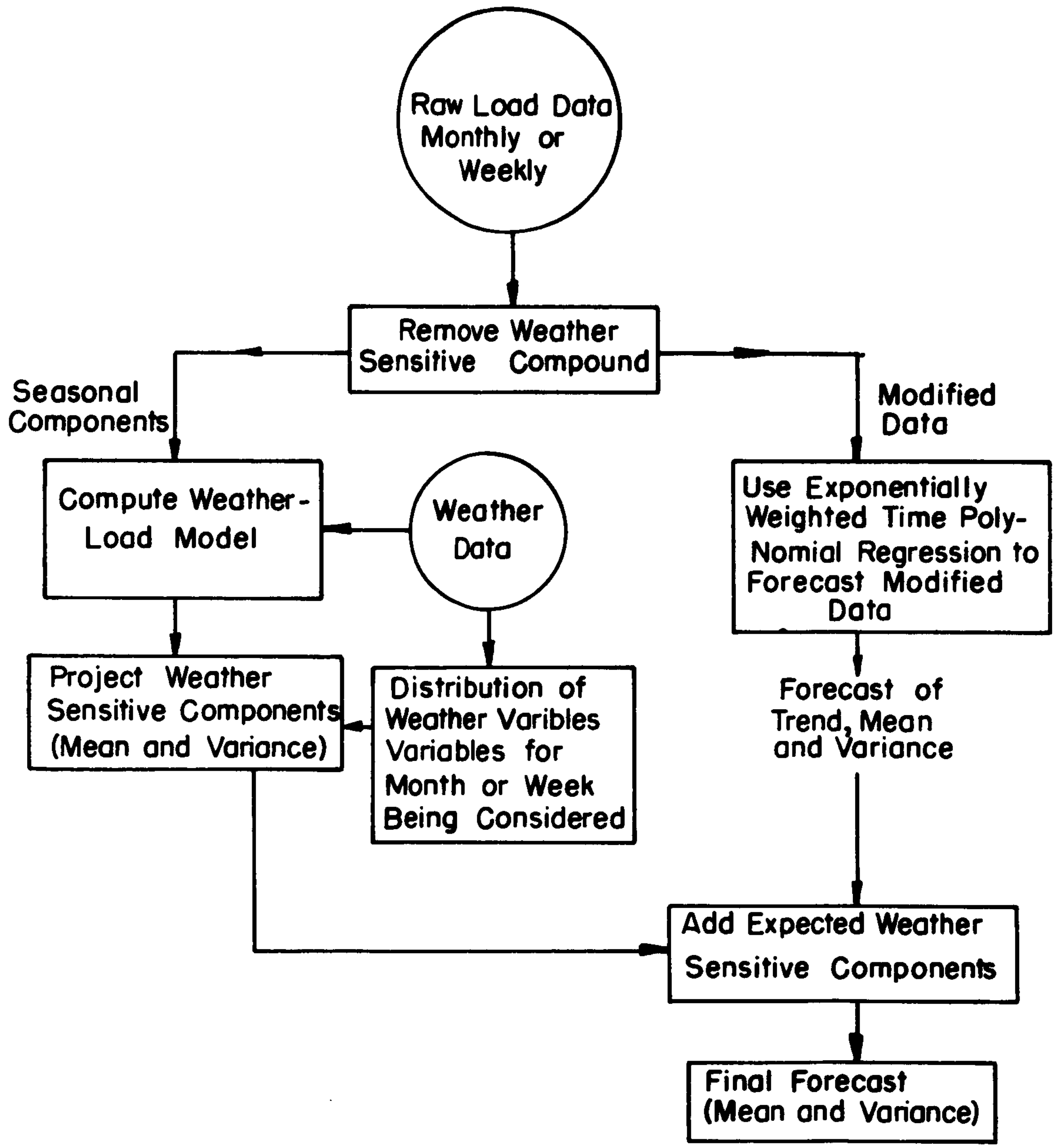

Fig.1. Forecasting Scheme

c. Distribution Circuit Forecasting

Transformer loads can be treated as a stochastic time series consisting of data arranged chronologically. Probability methods are useful for determining probable peak demand. Weather data, customer density, and weather sensitive loads strongly affect transformer loading and load growth. Processing of data for a number of distribution transformers that include customer load and weather data has been initiated to develop a load management and forecasting model.

B. THREE-PHASE DISTRIBUTION CIRCUIT ANALYSIS PROGRAM

A. H. El-Abiad M. Cheney

W. N. Keene D. Tarsi

The objectives of this project are to develop methods and prepare a package of programs for the analysis of unbalanced three-phase distribution circuits involving radial and loop configurations. Specifically, the package will do the following studies:

1. fault calculations
2. load flow
3. flicker analysis

These programs have the following advanced features:

1. The ability to represent inductive coupling between phases.
2. The ability to handle both types of unbalance; (a) unbalanced loading and (b) unbalanced circuits such as different size conductors for different phases or double and single phase circuits.

3. Ability to handle grounded and ungrounded circuits such as in Y/Δ, Δ/Δ and open delta banks.

4. Provision for the use of series as well as shunt capacitors.

5. Ability to handle boosters, bucks as well as voltage regulators.

6. Loads can be treated as constant Kw and power factor, or adjusted such that station currents have given values.

Program Description

The Distribution Analysis Package is a series of subroutines designed to perform load flow, short circuit, and flicker calculations on any distribution circuit having only one source. The package will handle radial and/or looped, wye and/or delta circuits. The package uses three-phase representation rather than sequence representation, and will handle

1. Isolators (Y⏚ - Y⏚ , Δ - Δ , Δ - Y⏚ , and Y - Δ)
2. Regulators
3. Bucks and Boosts
4. Capacitor Banks

The use of three-phase representation allows the package to represent both unbalance in the circuit and unbalance in the loading conditions. Another unique feature of this package is the ability to represent 1, 2, and 3-phase lines.

The package consists of 6 main parts. These are:

1. Input - reads and converts input data to a form usable by the other portions of the package.
2. ZBus - generates the bus impedance matrix which is used to represent

the network.

3. Edit - rearranges portions of data converted by Input, and also adjusts the loads if desired.
4. Load Flow - performs a load flow analysis of the network.
5. Short Circuit - performs a short circuit analysis of the network.
6. Flicker - performs a flicker analysis of the network.

Main Requirements

The package is written in Fortran-IV and takes advantage of the logical and complex arithmetic facilities of 7090/94 Fortran IV. This version of Fortran is comparable to level G and H Fortran contained in the IBM 360 operating system (OS/360). The program requires an input file, an output file, and two sequential work files for its execution. This will not present any problems. However, there is a practical requirement on core size. In order to represent a 50 bus system, 32 K words of memory are required (128 K bytes). This is due to the data requirements of 3-phase method of representation. More data is required to represent an unbalanced condition than a balanced one.

Data Requirements

The data for the package can be broken down into 3 main catagories -- these being data describing the substation, data describing the lines, and data describing the busses.

The data for the substation consists of:

1. The substation base KV
2. The circuit's KVA base

3. The substation voltage (120 V base)
4. Input currents to the network
5. Impedances to ground behind the substation (used for fault studies)
6. Substation type (Δ or Y)
7. Substation name

The data for the lines consists of :

1. Connection data
2. Configuration and phasing codes
3. Line length
4. Regulator, Buck, and Boost data

The bus data containes the following:

1. Name
2. Connected KVA and PF on each phase
3. Demand - Diversity factor for bus
4. Type of bus and load (Y or Δ)
5. Capacitor data

One card is required for each line or bus. The coding of these cards has been kept as simple and as straightforward as possible.

Program Status

The present status of all of the parts of the package is summarized in the following table:

	Written	Debugged	Tested	Documented	Converted to 360
Input	X	X	100%	80%	X
ZBus	X	X	100%	80%	X
Edit	X	X	80%	80%	X
Load Flow	X	X	90%	0%	X
S.C.	X	X	100%	0%	X
Flicker	X	X	0%	0%	No

Description of Test System

The test network used to check the program has the following features:

1. About 50 line sections and 30 buses.
2. Both loop and radial parts
3. Boosts and Regulators
4. The main circuit
5. Several loops involving 2-conductor lines.
6. Unbalanced loading
7. Static condenser banks.

C. DISTRIBUTION NETWORK ANALYSIS BY MEANS OF STATISTICAL THEORY*

Ingemar Andersson

1. Introduction

When calculating loading and voltage conditions in a retail distribution network, it is necessary to take into consideration the diversity in the loads; i.e., that the individual loads in the network vary more or less independently

* Anderson, Ingemar, "Rural Distribution Network Analysis By Means of Statistical Theory," Perec Report No. 21, November, 1966.

of each other and hence, that the simultaneous demand has always a maximum value which is less than the sum of the individual customers' peak demands.

This problem is mainly of a statistical nature, and therefore it may be handled by methods from the statistical theory.

2. Maximum Simultaneous Demand

It has been determined from measurements that an individual load during high load periods may be considered as having an approximately normal distribution in course of time. This means that the probability, S, that a load is less than a certain value, P_i , may be expressed by

$$S = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X_o} e^{-\frac{X^2}{2}} dx \tag{1}$$

where $X_o = \dfrac{P_i - \overline{P}_i}{\sigma_i}$

$\overline{P}_i$ = mean value for the load during high load periods

σ_i = standard deviation during high load periods

Due to smilarities in human living habits, we know that the individual loads are not totally independent in general. However, it may be reasonable to assume that they are so during high load periods; i.e., we can write for the standard deviation, σ , for the simultaneous demand

$$\sigma = \sqrt{\sum_{i=1}^{n} \sigma_i^2} \tag{2}$$

These assumptions lead to the following expression for the peak value of the simultaneous demand in the network:

$$P_{max} = \overline{P} + \sqrt{\sum_{i=1}^{n} (P_{i,max} - \overline{P}_i)^2} \qquad (3)$$

where $\overline{P}$ = mean value for simultaneous demand during high load periods

$P_{i,max}$ = peak value for individual load

$\overline{P}_i$ = mean value for individual load during high load periods.

If we assume that all the individual loads are in average equal, which may be a reasonable assumption for many networks serving similar customers, we obtain

$$P_{max} = k_1 \cdot w + k_2 \cdot \sqrt{w} \qquad (4)$$

where w = energy consumed in the network over a certain time period, and k_1, k_2 are constants.

This expression gives the maximum power demand in a network in terms only of energy consumed, which in fact is the only information we generally have of the loads.

3. Maximum Voltage Drops

The voltage drop from the source to a load point may, with sufficient accuracy, be written

$$u_k = \sum_{i=1}^{n} \frac{r_{ik} \cdot P_i + x_{ik} \cdot Q_i}{U_o^2} \qquad (5)$$

where $Z_{ik} = r_{ik} + j \cdot x_{ik}$ = elements in the Z-matrix for the network,

$P_i + j\, Q_i$ = complex power in the i^{th} load point.

Assuming $\frac{Q_i}{P_i}$ = Constant, we can write

$$u_k = \sum_{i=1}^{n} a_{ik} P_i \qquad (6)$$

where a_{ik} are constants.

Thus the voltage drop, u_k , can be considered a sum of statistical variables, $a_{ik} P_i$, and the standard deviation for u_k during high load periods is given by

$$\sigma_{uk} = \sqrt{\sum_{i=1}^{n} a_{ik}^2 \sigma_i^2} \qquad (7)$$

Making the same assumptions concerning the loads as were made before, the following expression may be derived

$$u_{k,max} = k_1 \sum_{i=1}^{n} a_{ik} w_i + k_2 \sqrt{\sum_{i=1}^{n} a_{ik}^2 w_i} \qquad (8)$$

This is an expression for the maximum voltage drop to the k^{th} load point in terms of individual energy consumptions, w_i , and the bus impedances for the network; k_1 and k_2 are the same constants used in Equation (4).

4. Standard Deviations In Voltages

Slow voltage variations and, consequently, the standard deviation for the voltage, have a decisive influence on the quality of an electric supply. It would therefore be desirable to have a method to calculate the voltage standard deviations in a network.

Assuming that the voltage in the source point is constant, Equation (7) gives the standard deviations in the load points during high load conditions.

In general, we obtain by introducing correlation coefficients, ρ_{ij}

$$\sigma_{u_k} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \, a_{ik} \, \sigma_i \, a_{jk} \, \sigma_j} \tag{9}$$

The standard deviations for the loads, σ_i , may be written

$$\sigma_i = \sqrt{b \cdot \bar{P}_i \, (P_{i,max} - \bar{P}_i)} \tag{10}$$

where b is a constant.

Equation (9) and (10) yields an expression from the voltage standard deviation in a load point in terms of peak and mean demands in the network.

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D. RELIABILITY ANALYSIS OF DISTRIBUTION CIRCUITS

K. N. Stanton J. Clarke

G. Moore R. Kellams

Three graduate students are working on this project; two on data analysis and a third on data preparation. Data collection is exceedingly tedious and time consuming, but we believe that this step is necessary for the project to be successful.

From the standpoint of power system engineering, the project breaks new ground by seeking ways to use reliability in planning and design. The ultimate objective is a means of estimating the reliability of a distribution circuit before it is built. This is quite different from collecting outage data and calculating the reliability of some circuit after it is in operation. The key to the approach being adopted is an equation (model) which allows some useful measure of reliability (e.g., outage rate) to be calculated (estimated might be a better word) from an equation involving known parameters about the system design and the environment. This equation is probabilistic in nature, and its parameters can only be determined by statistical methods; hence our interest in collecting and processing data.

Objectives

1. Develop a realistic yet realizable model for the reliability of distribution network elements which includes environmental effects.
2. Provide techniques for utilizing outage data as well as design and environmental data to evaluate model parameters.
3. Provide a detailed example showing the utilization of outage data to de-

termine the model parameters.

4. Provide guide lines for data requirements, collection, and storage.

Present Status

A model based strictly on conditional probability theory has been developed. A pilot study which limits the systems under consideration to distribution circuits in the 10 to 15 KV voltage range is proceeding as data is collected. Currently, data from two companies, one representing a centralized urban system and the other representing a distributed urban and rural system, is being analyzed. Specific goals in this pilot study are to determine:

1. which environmental factors are both observable and significant, and which may be neglected either due to non-observability or insignificance, and
2. methods for utilizing available data in determination of the parameters required for the model.

As work progresses, other models will be considered, e.g., models based on assumed distributions.

Decision theory is being investigated, primarily as a means for choosing from among the various models and for extending the currently available distribution network reliability techniques by incorporating "customer worth" into the analysis.

Date Requirements

Data is currently being collected and processed for the pilot study. The basic data needed is

1. Outage data including as much detail as possible concerning actual

failure mechanism and location.

2. Environmental data which may be correlated with the outage data.

Environmental data is scarce and some rather unusual sources are being considered. One important requirement is a tree survey giving the number of trees under or adjacent to a line. This kind of information is very useful for scheduling tree trimming, and some companies have it, some do not. The same kind of information can be read off aerial photographs, and the Civil Engineering School has been most helpful in explaining and demonstrating the possibilities of aerial photography. Activity levels are important, particularly traffic, and this leads to a consideration of the position of distribution poles relative to thoroughfares, data on traffic accidents, etc. Weather is important, particularly wind, rain, and lightning, but its affect is strongly dependent on the presence of other environmental factors. For example, high wind may not be very troublesome on lines which are well removed from trees.

The data collected to date indicates basic incompatibilities with respect to reliability studies between the outage data and the environmental data as they exist in company files. Very logically, the former is generally recorded with respect to a particular circuit, whereas the existing available environmental data is with reference to a geographical grid system. The correlation of the circuit system and the grid system, while possible, greatly magnifies the labor involved in reliability studies. This problem has yet to be solved.

E. METHODS OF GENERATION SCHEDULING

A. H. El-Abiad

Y. P. Dusonchet

The current objectives of this project are to review all available scheduling methods and develop techniques that will form the basis of computer programs capable of performing one or more of the following scheduling tasks depending on the application requirements:

1. Optimum power generation scheduling
2. Optimum voltage, reactive power, and transformer tap scheduling
3. Incremental economic loss due to capacity limitations of dispatchable quantities.

Project Status:

1. An internal report "Study of Economic Operation of Power Systems - Part I - Economic Factors, Part II - Optimization Theory and Scheduling Techniques" has been prepared. This report was written by students as part of their graduate training and would require considerable effort for editing in order to be suitable for distribution.
2. Three papers on optimization theory have been translated from Russian and produced as a special PEREC report.
3. A technical report on "Methods of Generation Scheduling" is very close to completion and should be available for distribution early in 1967.
4. A new general method for accurate economic dispatch of both active and reactive generation has been developed. Because of the necessity for generality and accuracy, this method is too slow in terms of computa-

tion time for on-line application, but it is highly desirable for the following reasons:

a. To recommend economic operating voltage levels at different load levels and seasons.

b. To check periodically on the accuracies of the loss formula used by the automatic load dispatcher, and give an economic method to decide when a new loss formula is needed.

c. To make an accurate evaluation of prevailing penalty factors, and devise a method to use these accurate penalty factors to update the loss formulat in use.

d. To plan studies, especially in the case of comparing alternative generation plans from the point of view of operating costs.

We would like to program this method in order to evaluate its practical use and make the necessary recommendations to produce a working program. Work in this direction may be started during the Spring semester, depending on the availability of manpower and time, and the priority of other projects.

F. POWER PLANT AND POWER SYSTEM DYNAMICS

K. N. Stanton　　R. D. Gustafson*

T. J. Williams　　R. C. Walters*

S. Talukdar

a. Power Plant Dynamics

Two projects are nearing completion. The first is a comparative study of alternative methods for simulation of a drum type boiler. The second is a report which considers several boiler models, the assumptions behind these

* Mechanical Engineering, Purdue University.

models, and the data required to use them for simulation. Since both these reports will be available, at least in draft from, before the end of December, further details will not be included in this report.

b. Power System Dynamics

This project is an initial attempt to simulate power system transients following a large loss of generating capacity. We wish to plot the decay of frequency in the system taking the frequency characteristics of loads as well as the load pick-up capability of boilers into account. Several important simplifying assumptions are used in order to prevent the project from becoming too complicated too soon.

Conventional stability studies investigate the dynamic response of a power system for at most a few seconds after a disturbance occurs. To extend the period of simulation to several minutes (i.e., until the system has reached a new steady state operating condition after a large sudden disturbance) and preserve even a pretence of accuracy, it is necessary to include models to account for the boiler responses.

Status

A program has been written and is presently in the debugging stages. The Z matrix is calculated by a separate program, and the initial conditions are obtained from a load flow study of the system in its pre-disturbance state. The major difficulty seems to be in making the output of these programs compatible with the input requirements of the simulation program.

G. THE IDENTIFICATION OF ENERGIZED SHIELDED CABLES

W. L. Weeks

B. McClure

Progress has been sporadic, due to personnel changes and availability. However, a new amplifier chain was built and tested. This amplifier chain consists of a Darlington differential amplifier followed by a low frequency amplifier having a gain of about 5000. This amplifier was tested with a specially designed probe. The system performed satisfactorily in one sense but is still not acceptable because a) a clamping device is still required b) an operator adjustment is required in order to null out the effects of high currents in the shield.

An amplifier based on a commercial operational amplifier was also constructed. This amplifier may be arranged with either a differential input or straight unbalanced input. A new non-clamping type unbalanced probe was designed and constructed; plans were made for other probes.

Also, a small sixty-cycle transistor oscillator has been "bread boarded." This oscillator will provide a check on the operation of the probe and amplifier in field operation.

The physical characteristics of the cable samples provided by the contract sponsors have been measured and recorded.

N67-33602

II. LONG RANGE RESEARCH

A. CALCULATION OF SHORT CIRCUITS ON UNTRANSPOSED NETWORKS*

A. H. El-Abiad

D. C. Tarsi

Models for the analysis of unbalanced three-phase networks have been developed and programmed. These models are matrix generalizations of single phase models used in the analysis of large linear systems.[1] The three-phase models are similar to the single phase models with the following exceptions:[2]

1. A network element is represented by a 3 x 3 impedance or admittance matrix.
2. A bus voltage or current is represented by a 3 x 1 vector.
3. The elements of the bus impedance or admittance matrix are 3 x 3 matrices.
4. In 1, 2, and 3 above, the three coordinates correspond to the three phases or any transform coordinates, such as symmetrical component, Clark's components, etc.

A systematic method of solution to all possible combinations of bolted bus faults on untransposed or unbalanced networks has been formulated. Linear transformation matrices coupled with the three-phase bus impedance matrix form the basis of this method.

A computer program written at the Purdue Energy Research and Education Center uses the transformation matrices in the solution of faults on unbalanced

* From a paper "Claculation of Short Circuits on Untransposed Networks," to be presented at the IEEE Winter Power Meeting, New York, January 29-February 3, 1967.

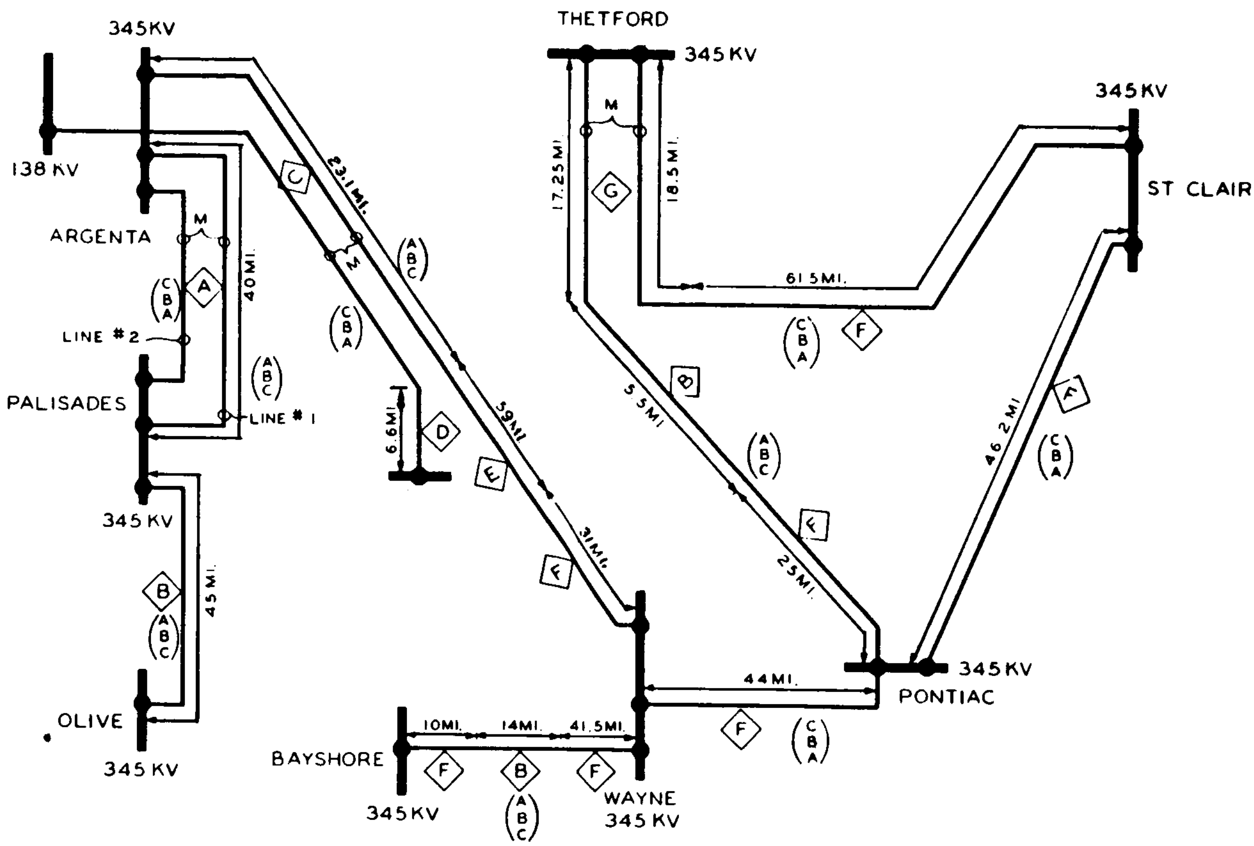


FIG. I SAMPLE SYSTEM

LEGEND

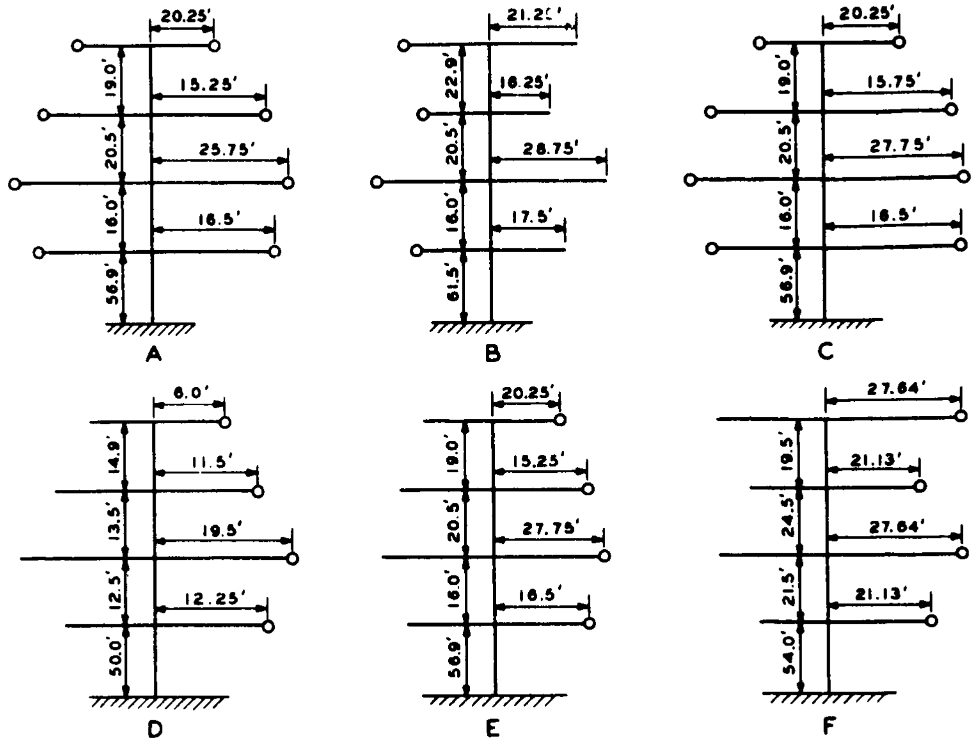
DENOTES CIRCUIT CONFIGURATION SHOWN IN FIGURE 2.

(A B C) DENOTES PHASING ARRANGEMENT PHASE (A) TOP PHASE, PHASE (B) MIDDLE PHASE, ETC.

M DENOTES MUTUAL COUPLING BETWEEN CIRCUITS.

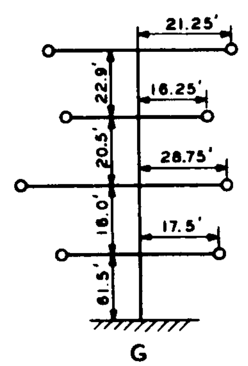
NOTE: THE PHASING WAS ARBITRARILY CHOSEN.

PROPOSED EHV SYSTEM FOR MICHIGAN'S POWER POOL



CONDUCTORS & GROUND WIRES

CONFIG	PHASE WIRES	GROUND WIRES
A	2156 MCM ACSR	7/16 STEEL
B	TWIN 954 MCM ACSR BUNDLED 18" SPACING	7/16 STEEL
C	LEFT SIDE - 795 MCM ACSR RIGHT SIDE - 2156 MCM ACSR	7/16 STEEL
D	795 MCM ACSR	3/8-7/S STEEL
E	2156 MCM ACSR	7/16 STEEL
F	TWIN 954 MCM ACSR BUNDLED 18" SPACING	159 MCM ACSR
G	TWIN 954 MCM ACSR BUNDLED 18" SPACING	7/16 STEEL



NOTES:

1. TOWERS ARE SYMMETRICAL.
2. SMALL CIRCLE ON END OF CROSSARM INDICATES PRESENCE OF PHASE OR GROUND WIRE.

CIRCUIT CONFIGURATIONS

FIGURE 2

three-phase networks. Figure 1 and Figure 2 show the proposed EHV network for Michigan's Power Pool, which was used as a test case.

By using this method, accurate short circuit analysis of all unbalanced three-phase networks can be easily programmed. Since all faults use the same method of approach, there is no longer a need for the different connections of the sequence networks for various types of bus faults. Similar methods have also been applied to the calculation of faults with impedances on untransposed networks.[3]

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B. ANALYSIS OF EXTRA LARGE SYSTEMS

A. H. El-Abiad

Y. Dusonshet

Methods and programs are under development that use tearing techniques to produce the solution of a large system by working on a part of the system (subsystem) at a time. For example, a power pool is divided into several subsystems, each represented by its bus-impedance-matrix, and the subsystems are tied together by tie-lines, each represented by its admittance. The programming system is being developed in steps. The first part to handle data storage and manipulation is complete, the second part to run load flow studies is 75% finished. Later other parts, such as generation scheduling, etc., will be added.

C. ANALYSIS OF EXTRA HIGH VOLTAGE SYSTEMS

A. H. El-Abiad D. Tarsi*

W. N. Keene

This investigation is directed towards the accurage modeling and simulation of untransposed EHV networks. A program to compute a three-phase ZBus matrix was developed. This matrix is used as the main model for short circuit and load flow analysis. The short circuit program is finished and is being presented in a paper at the IEEE Winter Power Meeting January 29-February 3, 1967. The program for load flow analysis is approaching completion and will constitute D. Tarsi's (Consumers Power Co.) Master's degree thesis. Work will continue in this area to evaluate the implications of superimposing the EHV systems on existing power systems and find solutions to any resulting problems.

D. ELECTRICAL TRANSIENTS IN POWER SYSTEMS

A. H. El-Abiad J. Villafane

R. Raghupathy A. Weiner

A coordinated study of electromagnetic phenomena on transmission lines was started this semester. Following is a list of this activity:

1. Propagation of electromagnetic energy on overhead transmission lines.
2. Traveling waves on transmission systems.
3. Laplace and similarity transforms for the solution of electric transients on transmission systems.

*Consumers Power Company

This area of research is newly established and will be pursued with the hope of developing several programs that will be valuable in investigating future EHV systems.

E. MEASUREMENT OF TURBO ALTERNATOR TRANSFER FUNCTIONS

K. N. Stanton

The turbogenerator, when connected to a large power system, can be treated as a self-contained subsystem with certain terminal properties. For dynamic representation, it is convenient to linearize the basic equation of the machine so that transfer functions can be used.

It has been possible to use normal operating data, measured at the generator terminals, to determine some transfer functions for the generator and its associated control systems. One interesting byproduct of this work is that it provides a means for estimating a composite figure for system and machine damping; a parameter that has not previously been measured under normal operating conditions.

Some work has also been carried out in studying non-linearities in the governor, but we are frustrated by the need for additional data.

N67-33603

III. CO-ORDINATION AND SUPPORTING ACTIVITIES

A. COMPUTER STORAGE AND RETRIEVAL OF BIBLIOGRAPHIC INFORMATION

K. N. Stanton R. Feng* B. Sommers*

A. H. El-Abiad D. Leite*

The PEREC bibliography on computer applications to power systems was dis-

*Computer Science

tributed in book form in April, 1965. It is presently being up-dated to include 1965 and 1966 references and a dictionary of keywords.

In addition computer programs and data files are being developed to automate the storage and retrieval of the material contained in the bibliography. This work, which can be regarded as a pilot project, will be complete by January, 1967.

A questionnaire has been prepared and sent to 60 engineering schools in the United States to determine the extent of research and educational activities in electrical power. The replies have been received and the findings will be summarized in a report. On the basis of this information it is hoped that increased coordination between engineering schools involved in research on power can be initiated.

PEREC is increasing its activities on the dissemination of information to the sponsoring companies. The general area of continuing education is also a natural part of the PEREC program, and investigations are underway to utilize new techniques of information handling more effective in dealing with this problem. Time sharing and data retrieval methods are one area of importance, and PEREC has a time sharing console connected with IBM Chicago using Quiktran. The possibility of writing programs in a conversation or interrogation mode is being given consideration. Man-machine interaction using real problems offers one new possibility for educating engineers in some areas. Increased use of seminars and engineer-in-residence programs are being considered.

I. ELECTROMECHANICAL ENERGY CONVERSION

E. M. Sabbagh L. Jenkins*

W. Shewan* R. P. Jetton*

Technological research may be inventive or developmental. Inventive research stems from the discovery of a new physical principle or from a new interpretation of an older law of physics. Developmental research progresses through the application of new material, the improvement of the properties of material, or through new modes of design. Sometimes the introduction of some new devices which affect the performance of a machine or the application of a new analytical approach can give stimulus to an area assumed dormant. In reality it is often difficult to draw a line between the areas of inventive and developmental research.

Since the development of the silicon-controlled rectifier (SCR) with its high current capability, its small size and its reliability, a new impetus has been given the area of electromechanical energy converters. New applications are opened for old devices. Thus a synchronous motor when controlled by SCR's may exhibit the characteristics of a d.c. motor, and react as a d.c. motor. Because of the ease of controlling their speeds, d.c. motors are extensively used in steel mills, paper mills and other types of industries, in trains and other means of transportation. Because of their commutators and carbon brushes, and also because of sparking, d.c. motors are not used in several applications where their performance characteristics are derived. Commutatorless a.c. motors exhibiting the characteristics of d.c. motors are sought. Furthermore our research has pointed the way to the possibility of a new kind of application which previously was not thought possible.

* Prof. Shewan, Head of EE Dept., Valparaiso University; Dr. L. Jenkins, Associate Prof. of EE, University of Louisville; Dr. R. P. Jetton, Assistant Prof. of EE, Bradley University, (Ill.).

As the currents in the silicon controlled electromechanical power converters are transient in nature, it was first necessary to find equations describing the instantaneous values of the currents and torques and to give their effective and average values, all in terms of speeds. This was done for the synchronous motor by Prof. Jenkins in a Ph.D. thesis submitted in June, 1965 to the faculty of Purdue University, entitled "Steady State Analysis of a Synchronous Motor With Periodically Interrupted Applied Voltages." Another phase of this technological research has been the use of silicon controlled rectifiers to the control of squirrel cage induction motors. The solution of this problem has been undertaken by Prof. Shewan, who has just completed this investigation, and is in the process of evaluating and writing the result of his research.

As the differential equations of both synchronous and induction motors cannot be solved without mathematical manipulations, Professors Jenkins and Shewan used the $y, \delta,$ and o and the instantaneous symmetrical components transformations to find the solutions to the instantaneous currents and torques in their respective machines. Prof. Jetton is applying a different analytical procedure -- namely, the Fourier series -- for the computation of the current and torque in an induction motor controlled by silicon rectifiers. All three investigators have used, or are using, digital computers to obtain numerical values for their solutions.

Since equations for the instantaneous currents and torques vs speed have been found by Jenkins and Shewan, it is now necessary to investigate the inverter circuits and find the ones which give the best performance for specific applications; for instance, the running of an induction motor at its maximum torque for different speeds while keeping a high efficiency.

SECTION 3.2

ELECTROMECHANICAL ENERGY CONVERSION

PROFESSORIAL STAFF

E. M. Sabbagh

(Assisted By)

Prof. Shewan - Valparaiso University
Assseociate Prof. L. Jenkins - University of Louisville
Assistant Prof. R. P. Jetton - Bradley University, Illinois

The field of application of our results encompasses the areas of self-propelled vehicles, from golf cars, automobiles, trains to satellites, as well as such other areas as paper mills, steel mills, washing machines, etc. In converting energy from heat directly into electricity (MHD, thermoelectricity, thermionic electricity, fuel cells, solar cells, etc.), the electrical power obtained is generally a d.c. power. Here we can use our SCR circuits as inverters and run our a.c. house appliances directly from the d.c. source while giving the motors of the appliances the characteristics needed for the job at hand.

The analytical work on the electromechanical energy conversion has been carried out without sponsors. For demonstration purposes Prof. Shewan has converted a small synchro into an induction motor, and has used some silicon rectifiers for control. The cost of this demonstrator is very small; however, further development in this area may be costly and require financial help. A demonstration of the performance of the device was given in the summer of 1966.

SECTION 4

ELECTROMAGNETIC FIELDS

PROFESSORIAL STAFF

C. L. Chen	S. B. Sample
C. M. Evans*	F. V. Schultz
R. A. Holmes	J. I. Smith*
D. B. Miller	W. L. Weeks

INSTRUCTORS AND GRADUATE ASSISTANTS

T. J. Gilmartin	R. K. Kaul
R. W. Graff	R. J. Lytle
R. T. Hilbish	G. M. Ruckgaber

C. L. Bennett

*On leave, 1966-1967
*Resigned, July 1, 1966

I. EFFECTS OF PLASMA ON ELECTROMAGNETIC WAVES AND DEVICES

A. WAVES GENERATED BY A DIPOLE IN A WARM, ANISOTROPIC PLASMA*

F. V. Schultz

R. W. Graff

The theoretical investigation of the waves produced by an electric depole in a warm, anisotropic, single-fluid, collisionless plasma, in which there is no bulk motion, has now been completed. The equations were reduced to a single inhomogeneous linear differential equation[1], a Green's function for the equation was proposed, and the resulting equation was Fourier-transformed and solved to produce an integral representation[2] for the field, requiring six integrations for the solution.

Five of these integrations were performed analytically, resulting in an exact closed-form integrand, which was then integrated numerically[3] for a specific set of the parameters which define the plasma, $\omega_p/\omega = 0.412$, $\omega_c/\omega = 0.750$, $U_e/c = 0.003$, and $\omega = 3.10^8$ radians/second, with the dipole oriented at 90° to the impressed magnetic field, $\bar{B}_o$. Here, ω_p is the plasma frequency, ω_c is the electron cyclotron frequency, U_e is the acoustic velocity in the electron gas, and c is the velocity of light in vacuum.

Field intensity diagrams were plotted in the plane of the dipole, using a radius of 10 meters, in the far-field region. Three modes resulted, roughly analogous to the ordinary, extraordinary, and electroacoustic waves found by the cold anisotropic analyses and warm isotropic analyses done by other writers.[4,5,6]

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*Research paritally sponsored by U.S. Army, Navy, and Air Force, Joint Services Electronics Program, Contract ONR N00016-66-C0076-A04.

Large maxima of radiation were found in the endfire direction which are not found by the cold-plasma analysis. In fact, most of the power seems to be radiated in the endfire direction, in a wave which is transverse as well as longitudinal, the rotation from longitudinal to transverse being produced by the impressed constant magnetic field, $\bar{B}_o$.

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B. A THEORETICAL DETERMINATION OF THE IMPEDANCE OF ANTENNAS IN A PLASMA THAT IS COMPRESSIBLE*

F. V. Schultz

R. J. Lytle

Numerical techniques for solving integral equations have been applied to finding the form of the current distribution on a linear antenna in a compressible isotropic plasma. The boundary conditions applied were (1) that the current be zero at the ends of the antenna, (2) that the normal component of fluid velocity be zero at the metal-plasma interface, and (3) that the tangential component of the electric field intensity be zero over the surface of the antenna, except at the gap in the center where the driving voltage is applied. Curves for the current distribution have been obtained for antenna half-lengths 1/4, 3/8, 1/2 acoustical wavelengths for operating frequencies of .1, .3, .9, 1.1, 1.5, 2, and 10 times the plasma frequency. Due to the nature of the results, the boundary conditions are being re-examined, and it appears that a mechanical body source term (which was neglected) may be significant. The problem is being reformulated to include this effect.

When the form of the current distribution has been determined, the radiated fields due to finite size linear antennas will be calculated, as will be the input impedance.

*Research partially sponsored by U. S. Army, Navy, and Air Force in the Joint Services Electronics Program, Contract ONR N00016-66-C0076-A04

C. HARMONIC GENERATION USING THE VARACTOR CAPABILITIES OF THE PLASMA SHEATH*

R. A. Holmes

R. T. Hilbish

Work has nearly reached completion on the problem, so a rather complete summary will be given.

Past efforts at harmonic generation using a plasma as the nonlinear media have used an r.f. discharge in which the driving source creates the plasma. There are two inherent disadvantages to this method; namely, loss of input power in creating the discharge, and the necessity of an impedance mismatch at the input. That the latter is true can be seen as follows. Suppose that an impedance match existed. Then, if for some reason the density of the discharge should decrease, a mismatch would occur resulting in a partial reflection of input power. Thus, the net power to the discharge would decrease, resulting in a further reduction in density and so forth until the discharge disappears. To bypass these difficulties, in this work a d.c. discharge is used with the restriction that no ionization by the input source be allowed.

Before consideration can be given to r.f. phenomena occurring in the plasma, a detailed knowledge of the d.c. characteristics of the discharge is necessary. In this work, coaxial geometry is used wherein a mercury discharge plasma is contained between two concentric metallic cylinders, which in turn are contained within a cylindrical glass envelope, as shown in Figure 1. A metal layer is wrapped around the outside of the glass to form a large coaxial capacitance with the molybdenum sheet cylinder. This capacitance is an r. f. short at the driving frequency. The r. f. source will

* Research partially sponsored by U.S. Army, Navy, and Air Force in the Joint Services Electronics Program, Contract ONR N00016-66-C0076-A04.

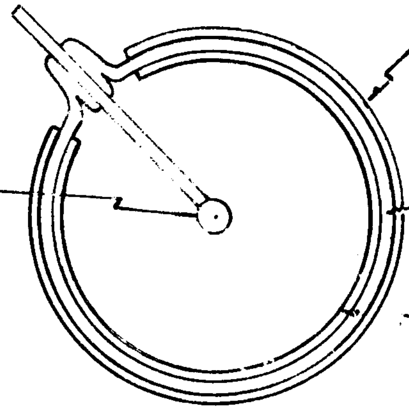

Fig. 1. End View of the Coaxial Cylinders

subsequently be applied between the innermost and outer cylinders, with the option of a d.c. voltage on the innermost cylinder (relative to the cathode of the discharge tube) while the outer cylinders form a d.c. block. A mercury pool spot discharge is used experimentally as the cathode for an auxiliary discharge along a 51cm long by 18mm diameter glass arm in which the above assembly is mounted, along with a Langmuir probe for measuring electron density and temperature.

In order to solve Poisson's equation for the d.c. potential distribution and consequently all other properties of interest, it is necessary to express the electron and ion density distributions in terms of the potential. For the electrons, the standard assumption of a Boltzmann velocity distribution throughout all space between the coaxial cylinders leads to the relation

$$N_e(r) = N_a \exp\left(eV(r)/kT_e\right), \tag{1}$$

where e is the coulomb charge, k is Boltzmann's constant, and Te the electron temperature, assumed constant throughout. Realizing that the potential distribution (assuming no applied positive bias) must be monotonically decreasing as

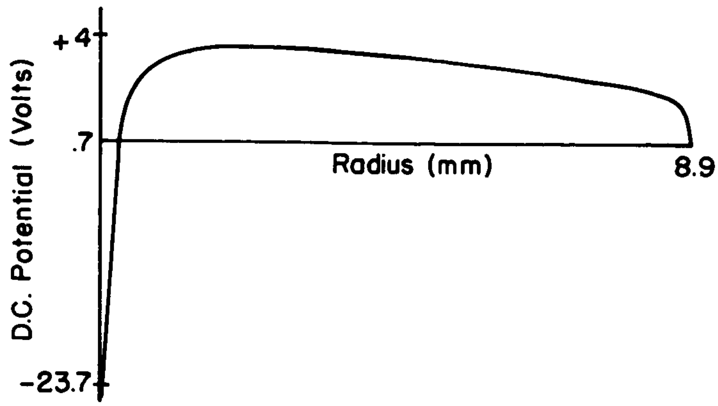

Fig. 2 Potential Distribution

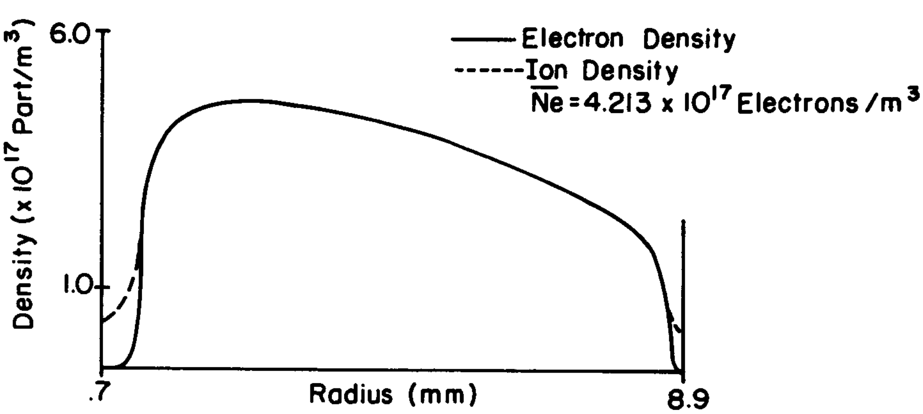

Fig. 3 Particle Density Distributions

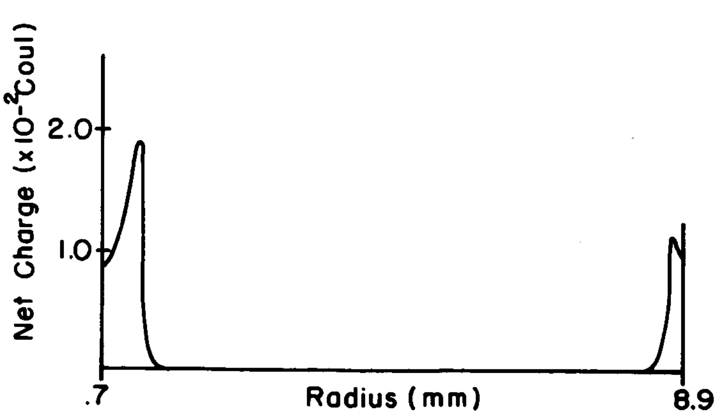

Fig. 4 Net Charge Distribution

both cylindrical walls are approached, (this rules out localized electron trapping) a potential minimum must occur somewhere in between. For convenience, the potential is defined to be zero at this point $r=r_a$. Thus, N_a is the corresponding electron density.

To obtain an expression for the ion density, it is assumed that ions are created by electron collision with neutral atoms at a rate proportional to the local electron density and with no initial velocity

$$S(r) = \nu_i N_e(r), \tag{2}$$

where ν_i is identified as the inelastic collision frequency. Those ions created in the region $r_c < r < r_a$, where r_c is the inner cylinder radius, will be accelerated toward the inner wall; those created in the region $r_a < r < r_\omega$, where r_ω is the outer wall radius, will be accelerated toward the outer wall. Thus, considering the region $r_c < r < r_\omega$, an ion created at some point r' will attain a velocity at some other point r,

$$v(r) = \left(\frac{2e}{M}\left(V(r) - V(r')\right)\right)^{1/2} \tag{3}$$

where M is the ion mass. The ion density in some small volume about the point r will be proportional to the number of ions created per unit time in the volume multiplied by the time they spend in the volume (or equivalently divided by their velocity). Assuming unit axial area, the total ion density of point r will be given by,

$$N_i(r) = \frac{1}{r}\int_r^{r_a} \frac{S(r')r'dr'}{\left(\frac{2e}{M}\left(V(r)-V(r')\right)\right)^{1/2}} . \tag{4}$$

A similar expression holds in the region $r_a < r < r_\omega$ except that the integration extends from r_a to some point r. Finally, writing Poisson's equation in cylindrical coordinates with radial dependence only, and substituting for the electron and ion densities as given by Eq. (1) and Eq. (4), the resulting equation for the potential is

$$\frac{d^2V(r)}{dr^2} + \frac{1}{r}\frac{dV(r)}{dr} = \frac{e}{\epsilon_o}\left[\pm\frac{1}{r}\int_r^{r_a}\frac{S(r')r'dr'}{\left(\frac{2e}{M}\,(V(r)-V(r'))\right)^{1/2}} - N_a e^{\frac{eV(r)}{kT_e}}\right], \tag{5}$$

where the plus sign is used for $r_c < r < r_a$; the minus for $r_a < r < r_\omega$, and ϵ_o is the permittivity of free space.

Solution of this equation is a relatively difficult numerical problem, the details of which are not discussed here. The result for the typical case of an average election density of $4.213 \times 10^{17}/m^3$ is shown in Fig. 2. The corresponding particle density distributions are shown in Fig. 3 and the net charge distribution in Fig. 4. In these figures a bias voltage of -23.5v has been applied to the inner cylinder. The radial dimensions correspond to those of the experimental tube.

With reference to the inner cylindrical region, three characteristic regions are noticeable: 1) a sheath region characterized by a negligible electron density and smoothly rising ion density (See Fig. 3), 2) a pre-sheath region characterized by a rapid rise in electron density, and 3) a uniform region in which charge neutrality is maintained to a considerable degree (Note in particular Fig. 4).

Suppose an r.f. voltage is applied across the plasma from the tungsten rod to the aluminum sheet of Figure 1. This r.f. voltage is assumed to have a frequency higher than the ion plasma frequency, yet much lower than the electron plasma frequency. Under this assumption the ions will not respond to the applied field and will remain in their d.c. distribution. The electrons will be strongly affected by the field, alternately drawn and repelled from the inner tungsten cylinder. In order to determine the significance of this motion, it is convenient to draw a piece-wise linear model of the particle distributions based on Figs. 2 and 3, as shown in Fig. 5.

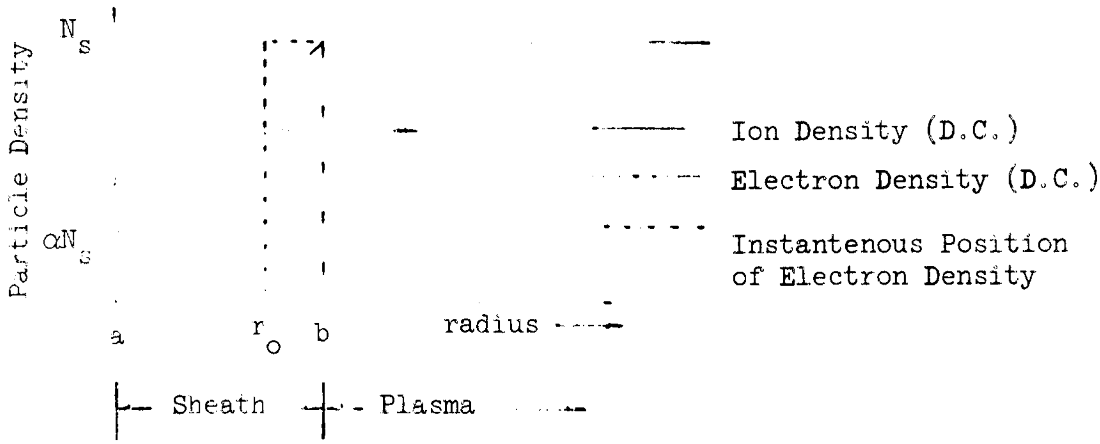

Fig. 5. Piece-Wise Linear Model of the Inner Sheath Region

The symbol α represents the fractional ion density reduction of the inner wall, r_o is the instantaneous electron density position, and N_s is the peak ion and electron densities, assumed equal in the plasma region. From the figure, the density distributions are found as,

$$N_e = \begin{cases} 0 & a < r < r_o \\ N_s & r_o < r \end{cases}$$

$$N_i = \begin{cases} \left[\left(\frac{1-\alpha}{b-a}\right) r + \left(\frac{\alpha b-a}{b-a}\right)\right] N_s & a < r < b \\ N_s . & b < r \end{cases} \tag{6}$$

Substitution into Poisson's equation, followed by a straight forward double integration with boundary conditions that both the electric field and potential be zero in the plasma region, leads to the result that the potential and electric fields at the inner wall are

$$V_a = -\frac{eN_s}{\epsilon_o}\left[A_1 + \frac{r_o^2}{2}\,\ell n\,\frac{r_o}{2} - \frac{r_o^2}{4}\right]$$
$$E_a = \frac{eN_s}{\epsilon_o a}\left[A_2 - \frac{r_o^2}{2}\right], \tag{7}$$

where A_1 and A_2 are constants involving a, b, and α. The corresponding charge on the inner wall is given by

$$Q_a = (2\pi a\ell)\epsilon_o\, E_a, \tag{8}$$

where ℓ is the axial length of the inner cylinder. An incremental capacitance can be defined as

$$C = \frac{dQ_a}{dV_a} = \frac{dQ_a/dr_o}{dV_a/dr_o} \quad ,$$

and upon taking the indicated derivatives from Eq. (7), one finds

$$C = \frac{2\pi\epsilon_o\ell}{\ell n(r_o/a)} \tag{9}$$

Note that this is just the equation for a coaxial capacitance which, however, is voltage dependent through the dependence of r_o on V_a in Eq. (7). Thus, Eq. (9) written in its voltage dependent force becomes

$$V_a = -\frac{eN_s}{\epsilon_o}\left[A_1 + \frac{a^2}{4}\, e^{\frac{4\pi\epsilon_o\ell}{C_1}} \left(\frac{4\pi\epsilon_o\ell}{C_1} - 1\right)\right] \tag{10}$$

A rather interesting result can be obtained by defining a potential V_m corresponding to complete collapse of the electrons into the inner wall. Thus, from Eq. (7) with r_o=a,

$$V_m = -\frac{eN_s}{\epsilon_o}\left[A_1 - \frac{a^2}{4}\right]$$

and Eq. (10) can be written

$$V_a - V_m = -\frac{eN_s a^2}{4\epsilon_o}\left[1 - e^{\frac{4\pi\epsilon_o\ell}{C_1}}\left(1 - \frac{4\pi\epsilon_o\ell}{C_1}\right)\right] \tag{11}$$

Letting $x=4\pi\epsilon_o \ell/C$, then, if the inequality $\min(C) \gg 4\pi\epsilon_o \ell$ holds, the term $e^x(1-x)$ can be expanded and truncated at two terms to give $e^x(1-x) \simeq 1-x^2/2$, so that Eq. (11) becomes

$$C_1 = \frac{K}{(V_m - V_a)^{1/2}} .$$

This equation is functionally identical to that of an abrupt junction varactor diode.

In addition to the sheath capacitance, there are three loss mechanisms involved when r.f. is driving the plasma. The first is a result of particle collisions in the bulk plasma region. This loss can be minimized by keeping the drive frequency well above the collision frequency. For a mercury discharge of density used in this work, the collision frequency is slightly above the ion plasma frequency, so that the above condition can be readily met; the resulting equivalent series resistance is on the order of 1 Ω. A second and less obvious source of r.f. energy loss is through a collisionless loss phenomena occurring in the pre-sheath region. Electrons entering from the plasma region "collide" with a moving potential gradient in the pre-sheath region which can be shown by a small signal analysis to result in the reflection of the electrons with a net energy gain (which is subsequently lost through collisions, loss to the walls, etc.). Fortunately, this loss turns out to be quite small when the source frequency is well below the plasma frequency, and therefore can be neglected. The final and most significant loss is electron collection by the inner wall when the net potential across the sheath is a minimum. Although this type of loss is negligible in a varactor diode, it can not be neglected in this case because of the significantly higher electron temperature (11,000-15,000°K) and hence greater conduction current. This loss can be simply formulated by starting with the standard equation for current flow to a cylindrical boundary, which, upon assuming a Boltzmann velocity distribution for the electrons, is given by

$$J_e = J_o\, e^{eV_a/kT_e}$$

where

$$J_o = eN_s \left(\frac{2\pi kT}{m}\right)^{1/2} .$$

The incremental conductance is obtained by differentiating with respect to voltage, yielding

$$g = G_o\, e^{eV_a/kT_e}$$

where　　　　(12)

$$G_o = N_s e^2 \left(\frac{2\pi}{mkT}\right)^{1/2} A,$$

and A is the area of the inner wall.

Having determined an equivalent circuit to represent r.f. phenomena occurring within the plasma, it is of interest to check the validity of the circuit by comparison with experiment. The simplest meeting point is a comparison of current waveforms as predicted by the equivalent circuit and observed via a sampling oscilloscope when a voltage drive is applied. This has been done, and excellent agreement found not only in waveform shape but also in dependence upon drive power, bias, and density. There was one exception. It was noticed that when the voltage switched from negative to positive under conditions in which an appreciable conduction current was flowing, an overshoot in the current was apparent which was not predicted from the equivalent circuit. This overshoot was attributed to the finite time required for electrons to be swept from the sheath region when the voltage reversed, a similar affect which occurs when a semiconductor diode is switched. The interpretation was verified by noting that the transient diminished as the bulk plasma density was increased. This is as expected since, for a given voltage across the sheath, the sheath becomes thinner as the density is increased, giving rise to a stronger electric field and therefore greater sweeping force. As the density increases, the plasma frequency increases, implying that the quasi-static assumption made in deriving the equivalent circuit is more closely approached.

Having explained and verified experimental phenomena occurring in a plasma contained between concentric cylinders under the influence of an applied r.f. voltage, and in particular noting the similarity to a varactor diode, it is of interest as an application of this device to find how efficiently second harmonic power can be generated. This has been done using a 143mc source at power levels up to a few watts, and standard varactor doubler circuitry. The maximum conversion efficiency was found to be approximately 31%. Although this number does not compare well with that obtained from a varactor diode in the same circuit (78.5%), there are a number of reasons which leave open the possibility of substantial improvement.

It is well known that the peak possible conversion efficiency for a doubler is a strong function of the driving frequency relative to the device cutoff frequency, defined as the inverse of the series equivalent RC product. Reducing the loss raises the cutoff frequency and also the maximum possible conversion efficiency. A very effective way to reduce the conduction loss in the plasma varactor is to use a ceasium plasma rather than mercury. In a ceasium discharge the electron temperature runs 5 to 10 times lower than that of mercury, which with reference to Eq. (12) results in a substantial reduction in conduction loss. The bulk plasma collisional loss can be reduced by increasing drive frequency.

It should also be noted that although the conversion efficiency of the plasma varactor may not in general approach that of a varactor diode, the plasma varactor with its much higher breakdown (arcing) voltage is capable of handling a considerable amount of power. This is particularly true at the higher frequencies where the varactor diode is quite limited.

D. INVESTIGATION OF NON-LINEAR, COLLISION AND COLLISIONLESS, PROCESSES IN PLASMA*

D. B. Miller

F. V. Schultz

The most interesting of current unexplored areas in plasma physics concern non-linear, collision and collisionless processes. It is the intent of this research program to verify and extend some of the theoretical treatments of these subjects by controlled experimental measurement.

Non-linearities are inherent in the Boltzmann equation describing the dynamic behavior of the distribution function, and analytical techniques for solution generally involve some sort of infinite series expansion.[1,2] The experimental problem then is to correlate experimentally measurable quantities, such as wave propagation and damping constants, with the coefficients of these theoretically assumed series terms.

Although an exact analysis of the collision process contains rather involved integrals, the problem is often theoretically simplified by use of an

*Research partially sponsored by U.S. Army, Navy, and Air Force in the Joint Services Electronics Program, Contract ONR N00016-66-C0076-A04.

effective collision frequency, ν. A perturbed distribution function relaxes back to equilibrium in the manner,

$$\sim \exp(-\nu t),$$

and ν can also be interpreted as the viscous coefficient in the electron equation of motion, so this collision frequency is closely allied to empirically determinable quantities: plasma dynamic response, wave damping, and conductivity, for instance. Careful measurement of the effective collision frequency, along with independent determination of other basic plasma parameters ought to shed light on the validity of this collision frequency concept.

Wave damping and plasma heating can occur under low density conditions, where one would expect ν to be negligibly small. The Landau damping phenomenon is one such collisionless energy exchange process which has been theoretically identified and experimentally demonstrated.[3] Another, concerned with cyclotron resonance in a magnetized plasma has not, however, received as thorough a study. Both its theoretical basis[4,5] and experimental verification,[5] while strongly indicative, are not yet as rigorous as one would desire. It would be another goal of these plasma experiments to provide a more unequivocal demonstration of this phenomenon.

This project is currently concerned with planning, procuring, and assembling an experimental facility in which the above defined measurements can be made. A thermal cesium plasma will be used since it gives a much "quieter," less energetic, and more controllable plasma than can be obtained in an electrical discharge. A coaxial cylinder geometry will be used, and plasma behavior will be measured by propagation of pulsed and c-w UHF electromagnetic waves down this coaxial transmission line. In addition, standard plasma diagnostics techniques, such as spectroscopy and Langmuir probes, will be used to yield independent measurement of plasma properties.

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N67-33605

II. SCATTERING OF ELECTROMAGNETIC WAVES BY SOLID OBJECTS*

The objectives of this research are:

1. To gain a better insight into the phenomenon of the scattering of electromagnetic waves that strike an object, the major dimensions of which are of the order of a wavelength.
2. To develop methods for calculating the magnitude and phase of the waves scattered by the above-mentioned object.
3. To apply the methods developed under 2, above, to the problem of determining the radar reflectivity of objects having shapes of practical interest.
4. To apply the knowledge gained to the problem of altering the radar cross-sections, or reflectivity, of objects of given volume.

This work has been in progress at Purdue since May, 1958, and has resulted in quite a number of papers, technical reports, and doctoral theses[1-7]. A perusal of these titles gives the background for the discussion of the work done during the past six months. This work is discussed in the following reports.

A. SCATTERING BY A FINITE CONE

F. V. Schultz

During the past six months only a rather limited amount of effort has been available for the development of improved methods of calculating numerical results for the radar cross-section, the near fields, and the surface currents

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on a perfectly conducting cone of finite height, which is struck nose-on by a uniform plane electromagnetic wave. Most of the effort has been applied to the probelem of evaluating the two integrals mentioned in the last semi-annual report. It has been found possible to obtain analytic expressions for the integrals over most of the range of integration, but numerical integration is required over the very small remaining range. Work is now going forward on developing computer programs for obtaining the required numerical results.

When these new programs are available, they will be used to calculate not only the near fields and the surface currents, but also to recalculate the far fields and radar cross sections, which were reported in TR-EE64-18, School of Electrical Engineering, Purdue University, September, 1964. It is hoped that the results will be valid for much higher values of ka than were the results given in the technical report above. Here $k = 2\pi/\lambda$, λ is the wavelenth, and a is the radius of the base of the cone.

B. SCATTERING BY A HEMISHPERE

F. V. Schultz

R. K. Kaul

Theoretical investigation of the scattering of a plane uniform wave by a perfectly conducting hemisphere and the back scattering radar cross section for both positive and negative z-axis incidence is now complete. Numerical computations for the radar cross section for values of KA ranging from 0.10 up to 50.0 have also been performed for both the directions of incidence.

Computations were performed first in the case of normal incidence on the flat base of the hemisphere,and the results were very encouraging, both qualitatively and quantitatively. Agreement with the $(KA)^4$ variation was almost exact in the Rayleigh region. In the physical optics region, even though the variation was somewhat oscillatory, the trend towards the ultimate $(KA)^2$ variation was clear, (as is to be expected as the flat base starts behaving like plane perfect reflector). The general appearance of the curve is that of smooth variation, indicating good convergence properties.

As a natural consequence, it was decided to carry out the computations for the reversed direction of incidence, the plane wave now striking the pole of the hemisphere normally. This involved recalculating the expression for the radar cross section, and making corresponding changes in the programs. Numerical results in this case, however, are not as satisfactory as one would wish. Again, the Rayleigh region results are almost identical with the previous case, but in the resonance region the curve fluctuates rapidly. The only qualitative result is that instead of rising steadily, as in the previous case, the curve does oscillate about the unity value, as one would expect since the object must behave as a perfect spherical reflector in the optics region.

It is suspected strongly that the convergence is much slower in this case, perhaps because the flat base part of the object continues to interfere with the spherical scattering characteristics up to much higher frequencies than vice-versa. Since the theoretical solution is not final, the machine storage is an obvious limitation. At the present time the program is set up for 40 equations (complex), and these do not seem to be adequate. It turns out that underflow and overflow are also limitations in a related manner.

In order to be able to overcome the storage problem, it has been decided to try to solve the problem afresh theoretically, modifying the coordinate system as suggested in the last report. If all analytical problems are overcome, the solution will enjoy the property of finality, and machine storage will cease to be a limitation.

C. INVESTIGATION OF RESONANCE REGION SCATTERING

F. V. Schultz

G. M. Ruckgaber

Work has terminated on the multipole matching method of determining plane wave electromagnetic scattering by axially-symmetric, perfectly-conducting targets in the resonance region, because of seemingly insurmountable numerical difficulties.

Excellent results can be obtained by this method if the surface of the scatter deviates only slightly from that of the sphere. For instance, if the ratio of minor-to-major axes is in the range 0.85 to 1.0, any unknown of the scattering problem can be found by using double precision arithmetic, with an accuracy of up to 15 significant figures for the sphere and for prolate and oblate spheroids. For spheroid axes ratios in the range 0.85 to 0.6, the accuracy of the field match at the scatterer surface (between actual matching points) decreases from 15 to 3 significant figures, and for ratios less than 0.5 the tangential scattered electric field actually diverges from the negative of the tangential incident electric field at the scatterer surface.

Numerically, the source of the problem is the spherical Hankel function, which fluctuates wildly on the scatterer surface if the surface is highly eccentric. With the system of simultaneous equations which must be solved to obtain the scattered field, these fluctuations account for a highly ill-conditioned coefficient matrix. Physically, the spherical vector multipoles cannot be easily matched to the incident field on a highly eccentric surface because of both the slow convergence of the series expansion of the scattered field on such a non-spherical surface and the associated numerical problems involved in determining the coefficients of such an expansion.

Because of its simplicity, the multipole matching method would be highly attractive for the solution of the scattering from an axially-symmetric scatterer whose surface is only slightly perturbed from that of the sphere. However, for highly eccentric scatterers (scatterers of more general interest) numerical problems require computational procedures seemingly beyond the ability of any digital computer system presently available.

D. GENERAL SCATTERING FROM THE FINITE CONE AND THE BI-CONICAL ANTENNA*

F. V. Schultz

G. M. Ruckgaber

Exact solutions for the scattering of a plane electromagnetic wave of arbitrary direction of incidence and polarization by the perfectly-conducting, spherically-capped finite cone and the bi-cone have been obtained. The forms of the solutions for the two scatterers are nearly identical. The space is sectionalized into two regions, and the electromagnetic field is expanded in terms of the spherical vector multipole functions of Stratton[1], as was done by Schultz, et. al.(2). Four doubly infinite expansion coefficients are used in each of the two regions. The boundary conditions are then used to obtain eight doubly infinite sets of equations in the eight doubly infinite sets of unknown expansion coefficients. Fortunately, four of the sets of equations are decoupled from the remaining four, resulting in two systems of equations of identical form.

These equations can then be solved numerically. Considerable computer time is involved, however, because the sets of unknowns are doubly infinite rather than singly infinite (the latter being the case with Schultz[1], et. al., where only "nose-on" incidence was considered). The Legendre eigenvalue constants μ_n^m, ν_n^m as defined by Schultz must also be determined here. However, for this more general problem it is necessary to evaluate the Legendre constants for all orders m of the Legendre functions. The problem is even more difficult in the case of the bi-cone, because the μ_n^m, ν_n^m are defined by the more complicated relations

$$\left.\frac{P^m_{\nu_n^m}(\cos\theta)}{Q^m_{\nu_n^m}(\cos\theta)}\right|_{\theta=\theta_1} = \left.\frac{P^m_{\nu_n^m}(\cos\theta)}{Q^m_{\nu_n^m}(\cos\theta)}\right|_{\theta=\theta_2}$$

$$\left.\frac{\dfrac{dP^m_{\mu_n^m}}{d\theta}}{\dfrac{dQ^m_{\mu_n^m}}{d\theta}}\right|_{\theta=\theta_1} = \left.\frac{\dfrac{dP^m_{\mu_n^m}}{d\theta}}{\dfrac{dQ^m_{\mu_n^m}}{d\theta}}\right|_{\theta=\theta_2}$$

for all m=0,1,2,..., n=1,2,..., where θ_1 and θ_2 are the bi-cone angles, and where P_ν^m, Q_ν^m are the associated Legendre functions of the first and second kinds.

Numerical work has not yet been undertaken because of the considerable cost involved. Nevertheless, numerical values of the scattered field and surface current for arbitrary direction of incidence for the finite cone would be of considerable interest, since the reciprocity theorem could then be applied to obtain the radiation pattern of arbitrary surface currents on the cone. This information would be of great value for radar cross section modification of nose cones, etc.

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E. GENERAL SCATTERING FROM THE FINITE CIRCULAR CYLINDER

F. V. Schultz

G. M. Ruckgaber

Work has been started towards determining the scattering of a plane electromagnetic wave by a finite, perfectly-conducting, circular cylinder. It is hoped that the space-sectionalization technique will yield a solution to this classical problem.

The simplifying case of a z-polarized broadside incident plane wave is being considered as a first attempt. If the space is sectionalized into five regions, the electromagnetic field expressed in terms of the cylindrical vector multipole functions of Stratton[1], and the boundary conditions applied, the result is twenty-two integral equations in terms of ten unknown expansion functions. The variable of integration in the integrals is the separation constant h resulting from the separation of variables technique used in the solution of the scalar Helmholtz equation. Since h can be complex, the path of integration in each of the integrals is unknown. It is hoped that restrictions on the value of h will become evident.

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F. MODIFICATION OF THE BACK SCATTERING CROSS-SECTION OF A METAL CYLINDER BY IMPEDANCE LEADING

C. L. Chen

The control of the scattering of an electromagnetic wave from a metal rod by center loading is of current interest. Existing theories[1] are based on the quasi-zero order solution of linear antennas, and therefore are valid for rods with half-length less than or equal to the half wavelength of the incident waves. Our investigation is directed to the longer rods.

The geometry of the problem is shown in Fig. 1. A perfectly conducting

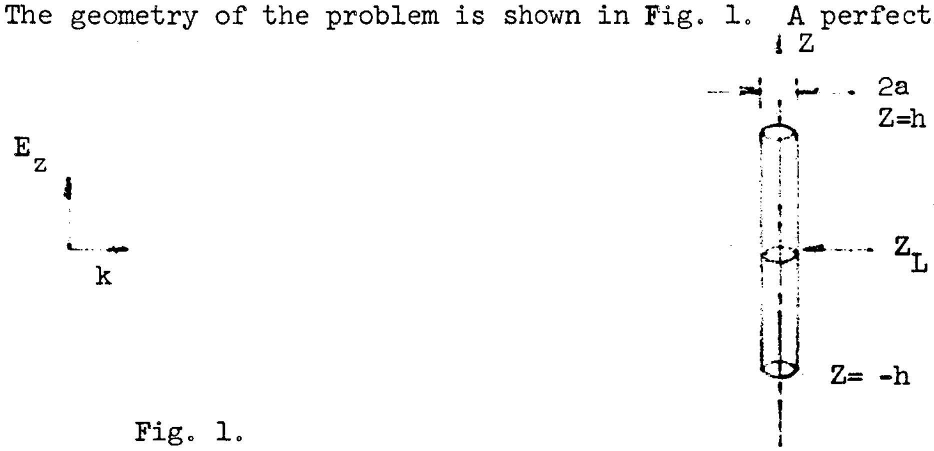

Fig. 1.

rod with radius a and half length h is illuminated by a plane wave of wavelength λ. The impedance loading Z_L is located at the center. If the radius a is much smaller than the wavelength, then the induced current is mainly along the axial direction, or z-axis. An integral equation can be found

$$\int_{h}^{h} I(z') K(z-z')dz' = \frac{i4\pi}{k\xi_o} E_o [1+C \cos kz - \frac{kI(0)}{2E_o} Z_L \sin k|z|]$$

$$; |z| < h \qquad (1)$$

where $\xi_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$, $k = \frac{2\pi}{\lambda}$ are, respectively, the characteristic impedance and the propagation constant of free space, $I(z)$ is the induced current, E_o is the strength of the incident wave, C is an unknown constant to be determined by the boundary condition at $z = \pm h$. A time harmonic variation of the form e^{-iwt} is assumed and suppressed. The Kernel $K(z)$ is given by

$$K(z) = \frac{1}{2\pi} \int_{o}^{2\pi} \frac{e^{ik\sqrt{z^2+4a^2\sin^2 \frac{\theta}{2}}}}{\sqrt{z^2+4a^2\sin^2 \frac{\theta}{2}}} d\theta \qquad (2)$$

For large kh, Eq. (1) can be approximated by

$$\int_{o}^{\infty} I(z_1')K(z_1-z_1')dz_1' = \frac{4\pi}{\xi_o k} E_o[1+c \text{ cak}(z_1-h) \ U\sin k|z_1-h|]H(2h-z_1)$$

$$z_1 \geq o \qquad (3)$$

where $H(x)$ is the Heaviside function, and

$$U = + \frac{kI(0)}{2E_o} Z_L \qquad (4)$$

Eq. (3) is an integral equation of the Wiener-Hopf type. In general, $I(z_1)$ is not bounded at the end $z_1=o$. However, if C satisfies an integral condition, then it is zero at the end. The Wiener-Hopf procedure is employed to find such an integral condition.[3] C is therefore determined, in terms of U,

$$C = - \frac{V(2h)-U \cos kh \ S(2h)+U \sin kh \ T(2h)-2U \sin kh \ T(h)+2U \cos kh \ S(h)}{\cos kh \ T(2h)+\sin kh \ S(2h)} \qquad (5)$$

In terms of C, and U, I(2) is found by Fourier transform technique

$$2\pi I(2) = \frac{i4\pi E_o}{k\xi_o} [\frac{\pi}{2\Omega_1} - i \frac{2\pi}{k} \{M(h-2)+M(h+2)\}$$

$$+ C\{\cos kz(\tfrac{1}{2}U(h-2)+\tfrac{1}{2}U(h-z))$$

$$+ \sin kz \ (S(h+2)-S(h-2))\}$$

$$+ U\{\cos kz(S(h-2)-S(2))$$

$$+ \sin kz \ (\tfrac{1}{2}U(h-2)+\tfrac{1}{2}U(2))\}$$

$$+ U\{\cos kz(S(h+2)-S(2))$$

$$+ \sin kz \ (-\tfrac{1}{2}U(h+2)+\tfrac{1}{2}U(2))\}] \qquad (6)$$

where Ω_1 is a function of ka.

Eqs. (4), (5), and (6) can now be used to solve for C and I(0), and hence U if Z_L is known. With C and U determined, the normalized back scattering cross-section σ_B/λ^2 is given by

$$\frac{\sigma_B}{\lambda^2} = \frac{1}{\pi|\Omega_1|^2} \left| kh+C \sin kh+U(Cp\ kh-1)\right|^2 \qquad (7)$$

The re-radiation pattern of the rod is found in a similar fashion.

In Eqs. (5), (6), and (7) functions $S(\bar{X})$, $T(\bar{X})$, $V(\bar{X})$, $M(\bar{X})$ are introduced for convenience. The exact integral forms of these functions are known. For large $k\bar{X}$, asymptotic expressions for these functions are obtained and found to be quite accurate. Nevertheless, numerical techniques can be used whenever greater accuracy is required.

This completes the theoretical analysis for the case of normal incidence. Numerical computation for σ_B/λ^2 is in progress.

It can be seen from Eq. (7) that for large kh, the major contribution to the back scattering cross-section is the term kh. Our aim, then, is to find an optimum loading Z_L to reduce this term, rather than reducing σ_B to zero. This optimum loading Z_L is then insensitive to frequency variation.

The analysis for the case of oblique incidence is also contemplated.

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III. PROPAGATION OF A LIGHT BEAM IN A RANDOM MEDIUM

N67-33606

F. V. Schultz

T. J. Gilmartin

The design of communication and telemetry systems which use the atmosphere as a transmitting medium requires knowledge of the relationship between the statistical properties of the turbulent medium and those of the received signal. This problem has been studied extensively by considering (1) the turbulent or random portion of the medium to be localized far from the transmitter or receiver[1], or (2) the signal to be an infinite plane wave[2], or (3) the signal to be a single ray[3]. In the practical case the beam is a finite-width electromagnetic wave with a non-uniform amplitude profile which undergoes diffraction, and propagates within an inhomogeneous medium over most of its path.

This project has as its purposes the determination of (1) the intensity profile of the beam at some distance into the perturbed medium, (2) the statistical properties of the amplitude, phase, and intensity fluctuations of the received signal, and (3) the spectrum of phase and intensity fluctuations in the practical case described above. A thoroughgoing analysis of the physics of the medium is not being attempted. A Gaussian correlation function for refractive index fluctuations is being used. The alternative correlation functions will also be used if it is decided that this would yield further information.

The intensity profile has been derived by using the first Born approximated solution to the wave equation, and numerical results are now being calculated. A zero-th order solution which describes the unperturbed, aperture-formed beam has been found; this solution is valid everywhere except in the practically negligible nearest portion of the near field. The solution originally proposed, which was valid only in the near field, has been shown to be a special case of the more general solution now being used.

The statistical properties which seem useful and are to be calculated as a function of position in the transverse receiving plane are: (1) the mean square phase fluctuation, correlation function, and correlation length, (2) the mean square amplitude fluctuation and correlation length, and (3) the mean square intensity fluctuation, correlation function, and correlation length. The derivations of general expressions for these quantities are essentially complete, and their numerical calculations are in process at this time.

Several approaches to the problem of determining the spectrum of intensity and phase fluctuations in the received signal have proven in applicable in this case. Other methods are being tried.

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N67-33607

IV. ELECTROHYDRODYNAMIC GENERATION OF CHARGED PARTICLE BEAMS

S. B. Sample

The electrical spraying of liquids from capillary tips is being studied in an effort to develop techniques for generating beams of extremely uniform microscopic charged particles. It is expected that these particle beams will prove useful in the study and control of the surface properties of solids, and will provide means for accurately controlling the deposition of thin films on solid substrates.

At present, a theoretical analysis of the equilibrium shape and characteristic frequencies of an electrically stressed liquid meniscus is being carried out. This analysis is based on an earlier theoretical treatment[1] of the static and dynamic behavior of liquid drops in electric fields. It is expected that the present work will make it possible to predict under what conditions the electrical spraying process becomes periodic (i.e., under what conditions the emitted particles are uniform in mass and charge).

In addition to the theoretical analysis, equipment is presently being acquired for an experimental study of electrical spraying from capillaries under the influence of intense time-varying electric fields. Initially, these studies will be performed with water in an air environment, and will be intended to check the theoretically predicted characteristic frequencies of the meniscus at the capillary tip. Later, experiments will be primarily concerned with the electrohydrodynamic generation of beams of liquid metal particles (e.g. aluminum, lead, gallium), and with developing methods for achieving uniformity in the charge and mass of the emitted particles.

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SECTION 5.1

COMMUNICATION SCIENCES

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N67-33608

I. ADAPTIVE AND LEARNING SYSTEMS

A. PARAMETER ESTIMATION WITH UNKNOWN SYMBOL SYNCHRONIZATION *

J. C. Hancock

T. L. Stewart

This report is a final summary of the investigation into signal parameter estimation when the signal starting times are unknown. The model for the problem was presented in a previous research summary.[1]

Restrictions on the problem are the following:

1. Known family of signal and noise densities
2. Known signal rate
3. No intersymbol interference
4. Independent signal classes and noise samples.

The following cases have been considered:

1. Estimation of signal and synchronization for periodic arrival times
2. Estimation of synchronization for periodic arrival times
3. Estimation of signal parameters for random arrival times.

The question of identifiability has been considered for cases 1 and 2. These results specify when the synchronization position can be determined from the data sequence. Also, the study of identifiability led to several extensions of previous theorems on identifiability for multi-dimensional gaussian, exponential, and translational densities.

The complete results of this investigation will appear in a future School of Electrical Enginneering, Purdue University, technical report entitled "Parameter Estimation With Unknown Symbol Synchronization."

*Supported by National Science Foundation, GP-2898.

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B. UNSUPERVISED LEARNING USING MOMENT ESTIMATORS

E. A. Patrick

G. Carayannopoulos

The application of the histogram concept to the unsupervised learning problem has been shown to involve a mixture of multinomial distributions.[1] In this work we exhibit moment estimators for the parameters of a mixture of two multinomial distributions.

Let there be two pattern sources ω_1 and ω_2 in a decision space Ω and let X_s be the s^{th} observation in an ℓ-dimensional space V^{ℓ}, where X_s is itself a sequence of v vectors: $X_s = \{X_{s_1}, X_{s_2}, \ldots X_{s_v}\}$. If source ω_1 is active, the source conditional c.d.f. of X is $F(X|\omega_i)$ where X is a generic representation of X_s. Denote the source probabilities by $P(\omega_1) = Q$ and $P(\omega_2) = 1 - Q$. When $F(X|\omega_i)$ is unknown, and there is not known structure of $F(X|\omega_i)$, it is natural to take $F(X|\omega_i)$ to be a multinomial distribution, characterized by R bin probabilities $\{p_t^i\}_{t=1}^R$. The c.d.f. of X is then a mixture given by

$$F(X) = \sum_{i=1}^{2} P(\omega_i)F(X|\omega_i) \tag{1}$$

The unsupervised problem is simply that of learning the bin probabilities $\{p_t^i\}_{t=1}^R$, i = 1, 2, ...M and Q. An illustration of the quantized probability space is shown in Figure 1 for a one-dimensional example.

Let b_t be the t^{th} fixed bin in the ℓ-dimensional observation space, and p_t the amount of probability in b_t from the mixture c.d.f. F(X) (1). Then

$$p_t = Qp_t^1 + (1-Q)p_t^2 \tag{2}$$

In order to demonstrate convergence of the unsupervised learning system under consideration, it is sufficient to show that Q, $p_t^{\ 1}$, and $p_t^{\ 2}$ in relation (2) can be learned for any t.

Consider a single bin b_t and denote by λ the number of samples out of v samples that fall into bin b_t when source ω_1 is active. Then the probability that γ samples fall into b_t is given by

$$P_2[\lambda=\gamma|\omega_1] = \binom{v}{\gamma}(p_t^{\ 1})^\gamma \ (1-p_t^{\ 1})^{v-\gamma} \tag{3}$$

Similarly, when source ω_2 is active,

$$P_2[\lambda=\gamma|\omega_2] = \binom{v}{\gamma} \ (p_t^{\ 2})^\gamma \ (1-p_t^{\ 2})^{v-\gamma} \tag{4}$$

and the probability that γ samples fall into b_t is

$$P_2[\lambda=\gamma] = \binom{v}{\gamma} \left[Q(p_t^{\ 1})^\lambda \ (1-p_t^{\ 1})^{v-\lambda} + (1-Q)(p_t^{\ 2})^\lambda(1-p_t^{\ 2})^{v-\lambda}\right]$$

$$\gamma = 0,\ 1,\ 2,\ \ldots v \tag{5}$$

The probability density function in (5) is a mixture of two binomial density functions whose parameters can be learned provided that $v \geq 2M-1$,[2] where M is the number of pattern sources. The first, second, and third moments of r.v. λ are given respectively by:

$$E[\lambda] = v(p_t^{\ 1})Q + v(p_t^{\ 2})(1-Q) \tag{6}$$

$$E[\lambda^2] = v\ Q(p_t^{\ 1}) + (1-Q)(p_t^{\ 2}) \quad + v(v-1)\left[Q(p_t^{\ 1})^2 + (1-Q)(p_t^{\ 2})^2\right] \tag{7}$$

$$E[\lambda^3] = v\left[Q(p_t^{\ 1}) + (1-Q)(p_t^{\ 2})\right] + 3v(v-1)\left[Q(p_t^{\ 1})^2 + (1-Q)(p_t^{\ 2})^2\right]$$
$$+ v(v-1)(v-2)\left[Q(p_t^{\ 1})^3 + (1-Q)(p_t^{\ 2})^3\right] \tag{8}$$

Equations (6), (7), and (8) can be solved explicitly for $p_t^{\ 1}$, $p_t^{\ 2}$, and Q, obtaining

$$p_t^{\ 2} = \tfrac{1}{2}\left[S + [(S)^2 - 4T]^{\frac{1}{2}}\right] \tag{9}$$

$$p_t^{\ 1} = \tfrac{1}{2}\left[S\ S + [(S)^2 - 4T]^{\frac{1}{2}}\right] \tag{10}$$

$$Q = \frac{\left[E[\lambda] - v(p_t^{\ 2})\right]}{v[p_t^{\ 1} - p_t^{\ 2}]} \tag{11}$$

where

$$S \triangleq p_t^{\ 1} + p_t^{\ 2} = \frac{E[\lambda^3]-E[\lambda]+3v(E[\lambda]-E[\lambda^2])+(v-2)E[\lambda](E[\lambda]-E[\lambda^2])}{-[v(v-2)(E[\lambda]-E[\ ^2])+(v-1)v-2)E^2[\lambda]]} \tag{12}$$

$$T \triangleq p_t^{\ 1}p_t^{\ 2} = \frac{(v-1)E[\lambda]S + (E[\lambda] - E[\lambda^2])}{v(v-1)} \tag{13}$$

Estimators for $p_t^{\ 1}$, $p_t^{\ 2}$, and Q result when $E[\lambda]$, $E[\lambda^2]$, and $E[\lambda^3]$ in (12) and (13) are replaced by the respective sample moments. The estimators developed above are asymptotically unbiased.

The computer simulated average performance of the above estimators for

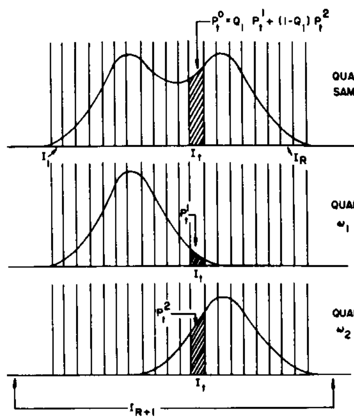

Fig. 1 Quantized spaces, I = 1, v = 1, M = 2

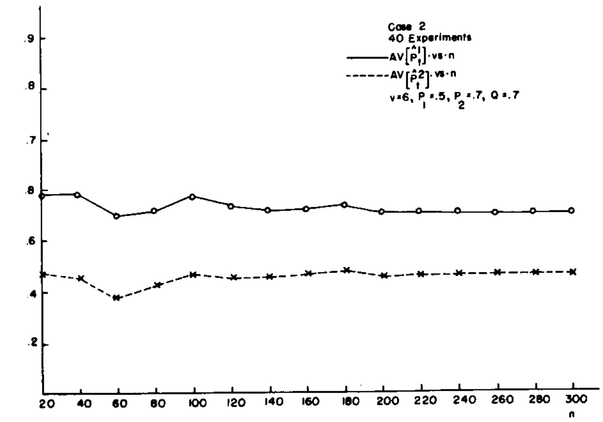

Fig. 2

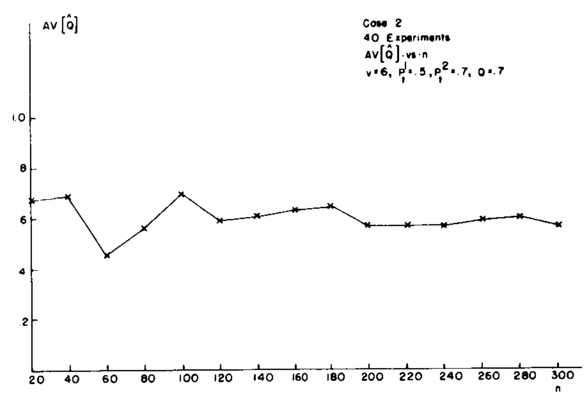

Fig. 3

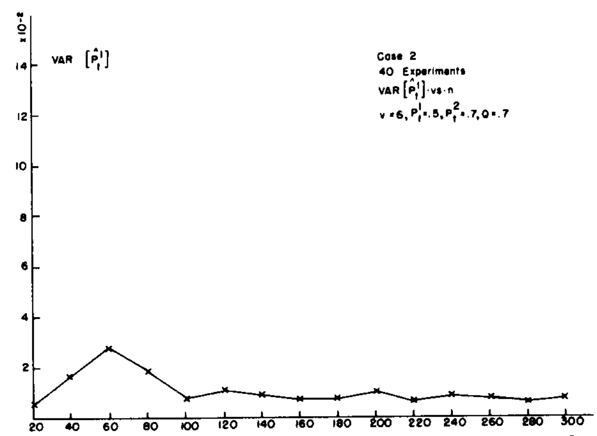

Fig. 4

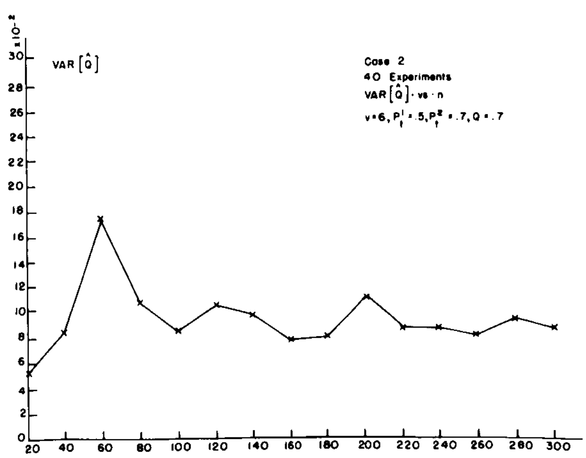

Fig. 6

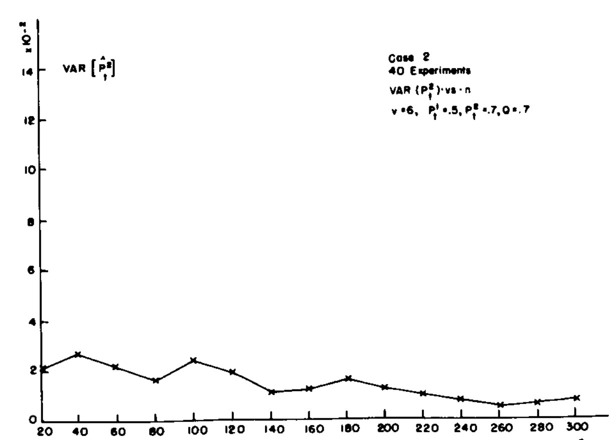

Fig. 5

v=6 is shown in Figures 2,3,4,5 and 6. The results demonstrate that unsupervised learning in an unknown stationary environment is possible.

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C. ANALOG IMPLEMENTATION OF AN UNSUPERVISED LEARNING COMMUNICATION SYSTEM

F. C. Monds

E. A. Patrick

A working model is being constructed to demonstrate the operation and performance of an unsupervised learning system for the signal detection problem shown in Figure 1. The problem concerns two randomly occurring classes of time

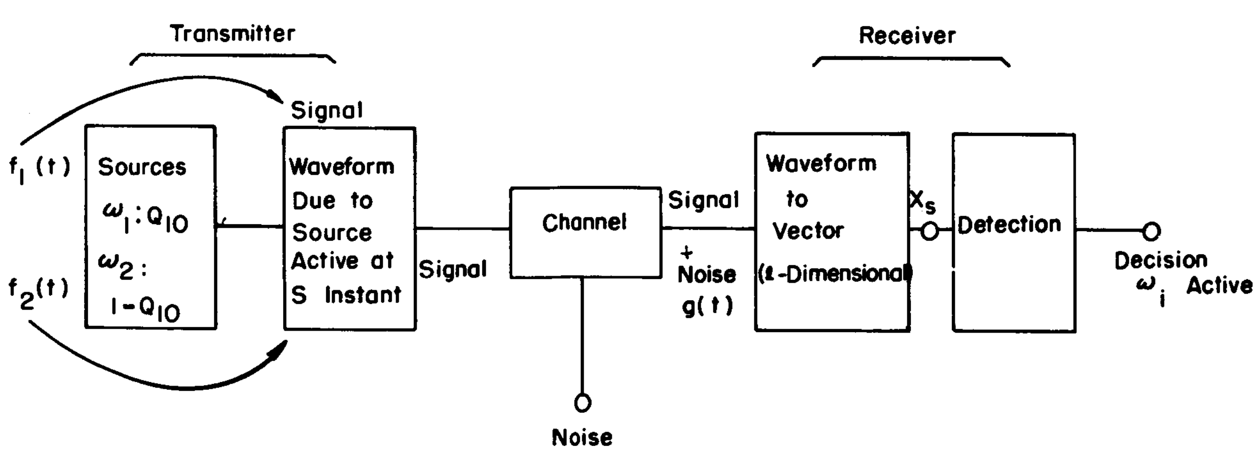

Fig. I System Model

functions indexed by ω_1 and ω_2 respectively. Class ω_1 occurs with probability P_1, and ω_2 with probability $(1-P_1)$. A sequence of n unclassified functions is generated and transmitted over a channel which adds noise to form $g_1(t), \ldots g_n(t)$ at the receiver input. The function $g_s(t)$ is processed to produce an ℓ-dimensional vector X_s. The distribution function of X_s, given ω_i active, is $F(X_s | \omega_i, \gamma_i)$, where $\gamma_i = E[X_s | \omega_i]$ and is unknown. $F(X_s | \omega_i, \gamma_i)$ is assumed gaussian with known covariance matrix, and P_1 is known also. The unknowns are thus the two ℓ-dimensional mean vectors $E[X_s | \omega_i] = \gamma_i$, i=1, 2.

The system being implemented uses a suboptimum approach, in which estimators $\hat{\gamma}_i$ are used for the fixed but unknown vectors γ_i.[1,2] Further simplification of the implementation is obtained by assuming synchronization between the transmitter and receiver so that any one X_s is caused by class ω_1 of class ω_2, but not by both. It has been shown[2] that "decision-directed" estimators for γ_i give a suboptimum system an asymptotic performance near that of a system with all parameters known. A decision-directed estimator is defined as a sample mean where the samples used have been classified by the receiver. The estimator is used to form the decision boundary, and is continuously updated during learning. When only one of the two mean vectors is unknown, the sample mean itself can be used as an estimator[3] and performs better than a decision-directed estimator in this case.

The analog model consists of a receiver and a signal generator module, each portable and self-contained. The mean vectors γ_i are each four-dimensional and can be arbitrarily selected. The source probability is variable. The added noise is white, and its variance σ^2 can be altered to give a wide range of chan-

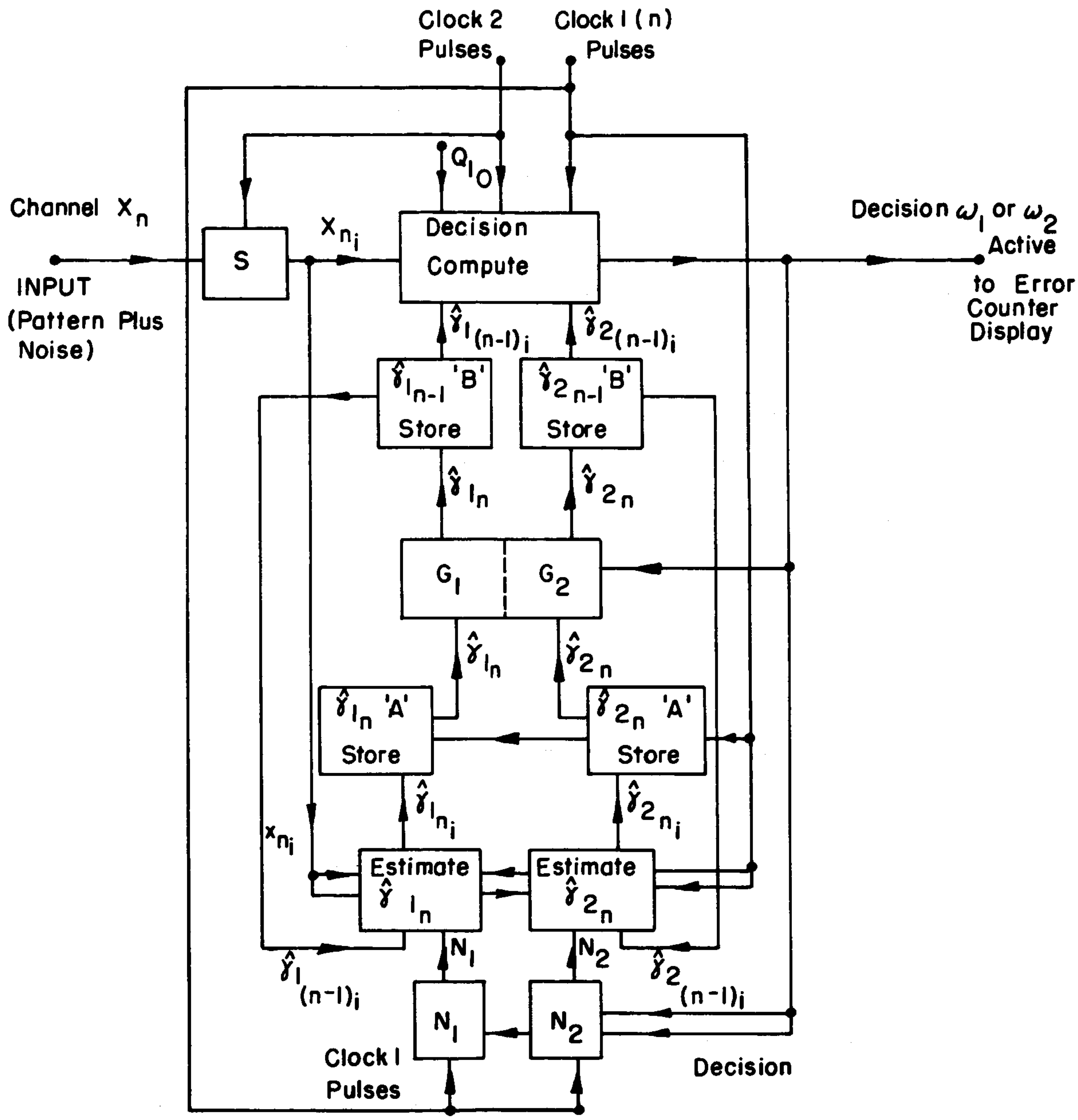

Fig. 2 Receiver Schematic.

Operation: One Iteration—Stage 1 – Computation of Decision, Involves Temporary Storage) and New Estimates. Stage 2—on Decision, Replacement of One ($\hat{\gamma}_{n-1}$) Estimate. by New ($\hat{\gamma}_n$) Estimate. Rejection of Other $\hat{\gamma}_n$

S – Sampling
G – Gate
N – Voltage Level Generator
($N_1 + N_2 = n$)

Pulse Repetition Frequency of Clock 2 = ℓ x Clock 1 P.R.F channel signal-to-noise ratio.

Figure 2 is a schematic of the receiver. The input signal is time-sampled and converted to pulse amplitude modulated form to give X_s. Multiplication and division in the required computations are performed by Hall-effect devices, and gated capacitor stores hold the continuously (during learning) updated mean vector estimates. The number of samples n used in the learning period can be varied. The receiver can also operate as a correlation detector (with known mean vectors) for performance comparisons to be made.

To date, the decision-making section (correlator) of the system, the signal generator module, and the counter display (to show the number of errors and experiments) have been completed and tested. The correlator gives a performance within 2dB(signal-to-noise ratio for a given probability of error) of that of an 'ideal' correlator. At present the estimate-making circuits are being tested.

The error count display makes it possible to easily compare the system performance for known mean vectors, decision-directed estimators and for sample mean estimators. The conditional probability of error at various stages of an experiment (each experiment consisting perhaps of a learning period) can be displayed.

The system could be applied to communications situations, for example binary pulse-code-modulation, where the transmitted signal alphabet is either unknown or is time varying. In the latter case, the system can have an adaptive feature in that the learning period can be altered to best match the rate of alphabet variation so that the error rate is minimized. It is intended that the final model will have this adaptive capability. Consideration is also being given to an extension of the system for more than four dimensions and more than two sources.

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D. DISTRIBUTION FREE, MINIMUM CONDITIONAL RISK LEARNING SYSTEM*

E. A. Patrick

A new Bayes approach to learning systems for unknown noise is developed with minimum risk conditioned on a "complexity constraint." By complexity constraint is meant that there is an upper bound on the number of fixed but unknown parameters that can be introduced. The risk also is conditioned on a set of structure; this set of structure is selected a priori from a family of sets. With the double constraint imposed by the complexity constraint and the set of structure, further structure is learned a posteriori.

Consistent with a Bayes approach, there are M pattern class $\{\omega_i\}_{i=1}^{M}$ in a decision space Ω. If ω_i is active at the s^{th} observation, then the ℓ-dimensional random vector $X_s \in I^\ell$ is observed with c.d.f. $F(X_s|\omega_i)$. A sequence of n such vectors $Y_n = X_1, X_2, \ldots, X_n$ is available (we will use X as a generic representation of X_s), and conditional risk is minimized in deciding which class caused X_{n+1}.

*This work supported by Air Force Contract AF 33(615)3768, Wright Patterson Air Force Base.

In reference 1, a Bayes solution to the unsupervised learning problem is given, and includes the problem where a functional form for $F(X|\omega_i)$ is unknown. For that problem, $F(X|\omega_i)$ is assumed a member of the family of multinomial c.d.f.'s, an application of the histogram concept to the unsupervised problem. As a multinomial c.d.f., $F(X|\omega_i)$ is characterized by R^ℓ parameters where R is the number of intervals or quantum levels in each of the ℓ dimensions. The multinomial assumption for $F(X|\omega_i)$ does not violate the true c.d.f. because it does not utilize structure. Unfortunately, R^ℓ can be large and the Bayes solution impractically complex. On the other hand, R^ℓ bins are not needed if provision is made for learning structure.

Let I^ℓ be composed of K disjoint regions I_t, $1 \leq t \leq K$ such that $I^\ell = \bigcup_{t=1}^{K} I_t$, $K = R^\ell$. For class ω_i, let these K regions be mapped to S_i regions $\mathcal{J}^i_\xi$, $1 \leq \xi \leq S_i$ where $\bigcup_{\xi=1}^{S_i} \mathcal{J}^i_\xi = I^\ell$ and $S_i \leq K$. The mapping is determined by the joint constraint (S_i, η^j_i) where η^j_i is a set of functions determining the S_i regions in the range. For M classes there are sets η^j_i, $1 \leq i \leq M$, where $\eta^j_i \in \eta^j$.

Only the supervised problem is considered (there are n_i samples distributed as $F(X|\omega_i)$, $1 \leq i \leq M$ -- the samples are classified). A basic assumption is that $\mathfrak{N}^j_i$ be a set of ordering functions for establishing multidimensional sample blocks or regions $\mathcal{J}^i_\xi$, $1 \leq \xi \leq S_i$, in I^ℓ. After defining structure, the minimum conditional risk solution with $F(X|\omega_i)$ a member of the family of multinomial c.d.f.'s is reviewed. This solution is then modified to be conditioned on S and $\mathfrak{N}^j$ where $S = \sum_{i=1}^{M} S_i$. Next a theorem relates conditional probability mass in I_t to that in $\mathcal{J}^i_\xi$ for each class ω_i. Then a theorem states that the conditional probability mass in $\mathcal{J}^i_\xi$ is a conditional expectation which is distribution free. Finally, a theorem states that to minimize conditional risk against S, η^j, and Y_n, observe the region $\mathcal{S}_\xi^i$ that $\mathbf{X}_{n+1}$ is in for each i and calculate the volumes of these regions. Decide class ω_a active if the volume of $\mathcal{S}_\xi^a$ is the smallest (assumes equal class probability but this is simply generalized). Next it is shown that if S = n, $n = n_1+n_2+\ldots+n_M$, the conditional risk converges in probability to the actual risk for any family of continuous c.d.f.'s, $F(\mathbf{X}|\omega_i)$, $1 \leq i \leq M$.

The practicality of the approach is demonstrated by the fact that a reasonable number of training samples n_i is $2\,\ell \leq n_i \leq 4\ell^2$ for η_i^j a set of linear functions. If $\ell = 10$, then $20 \leq n_i \leq 400$. Computer simulated performance verifies the merit of the new approach.

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E. DECISION DIRECTED ESTIMATOR FOR THE SOURCE PROBABILITY IN A BINARY UNSUPERVISED LEARNING SYSTEM

E. A. Patrick

G. Carayannopoulos

1. Introduction

Previous results when decision-directed estimators are used in place of fixed but unknown mean vectors assume that the source probabilities are known. This restriction is now removed by exhibiting a consistent estimator for the source probability. Computer simulated results on the performance of the three estimators are presented for the case where all three parameters are unknown.

A model of the problem under consideration is shown in Figure 1. One of the two sources ω_i, i = 1, 2, is active to cause the ℓ-dimensional vector $\mathbf{X}_s$ at the s^{th} observation; however, the active source is unknown. The c.d.f. of

X_s, given source ω_i is active, is $F(X_s|\omega_i, \gamma_i)$ where $\gamma_i = E[X_s|\omega_i]$. Denoting the source probabilities by $P(\omega_1) = Q$ and $P(\omega_2) = 1-Q$, it is assumed that γ_1, γ_2 and Q are unknown.

The problem is, given $X_1, X_2, \ldots, X_{n-1}$, decide which source caused X_n. It is not known which source caused $X_1, X_2, \ldots, X_{n-1}$. The suboptimum system we are considering substitutes "decision-directed" estimators for γ_1 and γ_2 and a moment estimator for Q.

The optimum decision equation (with γ_1, γ_2 and Q known) for deciding which source caused X_n is

$$(\gamma_1-\gamma_2)^t \; \Phi_{xx}^{-1}(X_n-X_o) > 0 : \omega_1 \tag{1}$$

where

$$X_o = \frac{\Phi_{xx}(\gamma_1-\gamma_2)}{(\gamma_1-\gamma_2)^t(\gamma_1-\gamma_2)} \ln \frac{1-Q}{Q} + \frac{1}{2}(\gamma_1+\gamma_2) \tag{2}$$

We now assume

$$\Phi_{xx} = \sigma^2 I$$

and γ_1 and γ_2 are estimated using "decision-directed" estimators defined by Eq.'s 2.2 and 2.3 in reference 1.

2. Moment Estimator For Q When γ_1 and γ_2 Are Known

The expected value of X_s over the sample space is

$$E[X_s] = Q\gamma_1 + (1-Q)\gamma_2 \tag{2.1}$$

If X_s is one dimensional, we can solve (2.2) for Q:

$$Q = \frac{E[X_s] - \gamma_2}{\gamma_1 - \gamma_2} \tag{2.2}$$

Replacing $E[X_s]$ by the sample mean

$$\bar{X}_s = \frac{1}{n} \sum_{s=1}^{n} \tag{2.3}$$

and γ_1 and γ_2 by the decision-directed estimators γ_1 and γ_2 in (2.2) gives the estimator $\hat{Q}$, for the source probability, for the case where γ_1 and γ_2 are unknown.

$$\hat{Q} = \frac{\overline{X}_s - \hat{\gamma}_2}{\hat{\gamma}_1 - \hat{\gamma}_2} \tag{2.4}$$

When X_s is an ℓ-dimensional, there are ℓ equations of the form (2.1):

$$E[X_{s_j}] = Q\gamma_{1j} + (1 - Q)\gamma_{2j}\ ,\ j = 1, 2, \ldots \ell \tag{2.5}$$

Since the expectation of the sum is the sum of the expectations,

$$E \sum_{j=1}^{\ell} X_{s_j} = \sum_{j=1}^{\ell} E[X_{s_j}] = Q \sum_{j=1}^{\ell} (\gamma_{1j} - \gamma_{2j}) + \sum_{j=1}^{\ell} \gamma_{2j} \tag{2.6}$$

Solving (2.6) for Q,

$$Q = \frac{\sum_{j=1}^{\ell} E[X_{sj}] - \sum_{j=1}^{\ell} \gamma_{2j}}{\sum_{j=1}^{\ell} (\gamma_{1j} - \gamma_{2j})} \tag{2.7}$$

An estimator for $Q(\hat{Q})$ results by replacing $E[X_{sj}]$ by $\overline{X}_j$ and γ_{1s}, γ_{2j}, by $\hat{\gamma}_{1j}$ $\hat{\gamma}_{2j}$, where $\overline{X}_j$ and $\hat{\gamma}_{1j}$, $\hat{\gamma}_{2j}$ are the jth component of the sample mean vector and the decision-directed estimates of the source are conditional mean vectors.

3. Statistical Properties of $\hat{Q}$ with γ_1, γ_2 Known

We shall examine the more general case where X_s is ℓ-dimensional; it is assumed that the covariance matrix is diagonal. Then the variance of $\hat{Q}$ is given by

$$\mathrm{Var}(\hat{Q}) = E\{[\hat{Q} - Q]^2\} = \frac{1}{b^2} E\{[\sum_{j=1}^{\ell} X_j - (\sum_{j=1}^{\ell} \gamma_{2j} + Qb)]^2\} \tag{3.1}$$

where

$$b \triangleq \sum_{j=1}^{\ell} (\gamma_{1j} - \gamma_{2j})$$

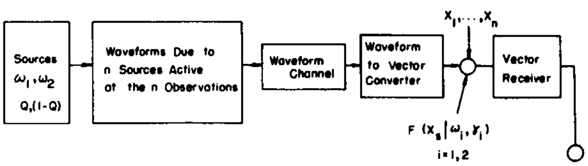

Figure 1 Problem Model

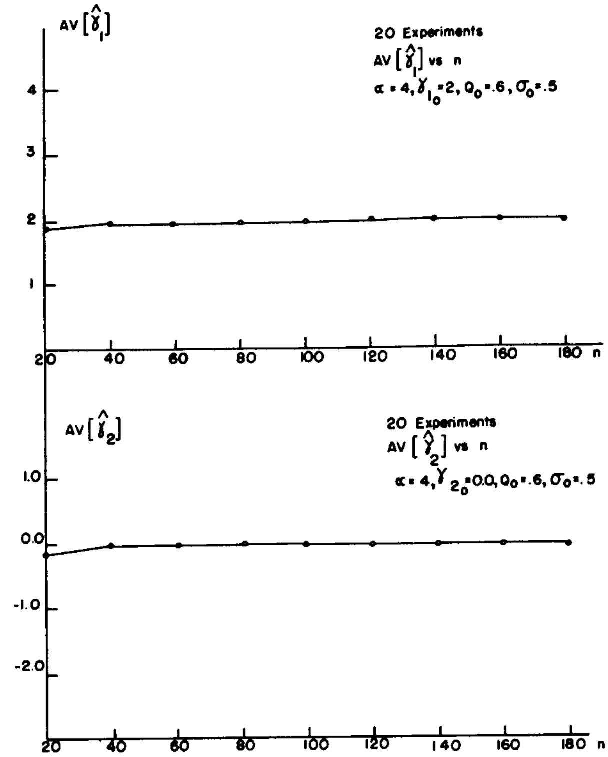

Fig. 2

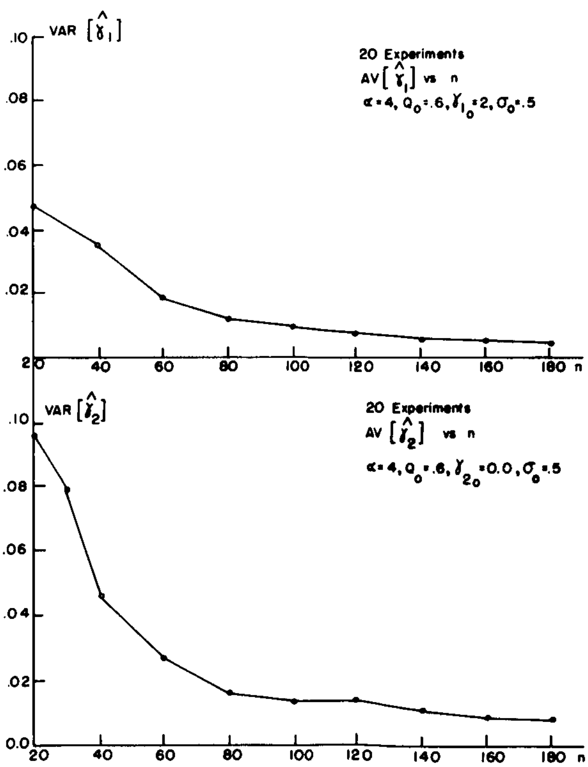

Fig. 3

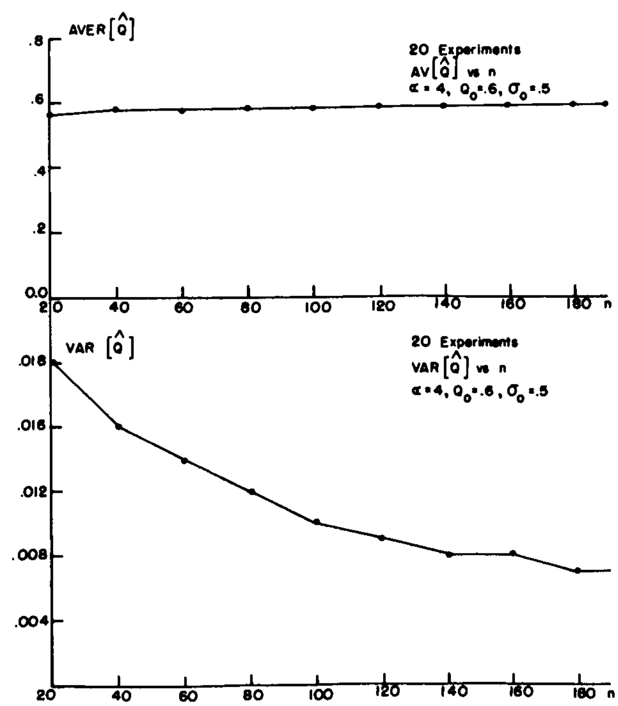

Fig. 4

and

$$\overline{X}_j = \frac{1}{n} \sum_{s=1}^{n} X_{sj}$$

the jth component of the sample mean vector expanding the square in (3.1) and carrying out the expectation, we obtain

$$\mathrm{Var}(\hat{Q}) = \frac{1}{b^2} \sum_{j=1}^{\ell} \frac{\sigma_j^2}{n}$$

if $\sigma_j = \sigma$ for $j = 1, 2, \ldots, \ell$

$$\mathrm{Var}\ (\hat{Q}) = \frac{\ell\sigma^2}{nb^2} = \frac{\ell}{n[\sum_{j=1}^{\ell} \alpha_j]^2} \tag{3.2}$$

where $\alpha_j \overset{\Delta}{=} \frac{|\gamma_{1j} - \gamma_{2j}|}{\sigma}$, the signal to noise ratio per dimension.

If $\alpha_j = \alpha$ for all j, then

$$\mathrm{Var}(\hat{Q}) = \frac{1}{n\ell\alpha^2} \tag{3.3}$$

From equations (3.2) or (3.3) we can make the following observations:

i) $\mathrm{Var}(\hat{Q}) \to 0$ as $n \to \infty$, thus $\hat{Q}$ is a consistent estimator for the source probability.

ii) $\mathrm{Var}(\hat{Q})$ is inversely proportional to ℓ, the number of dimensions.

iii) $\mathrm{Var}(\hat{Q}) \to \infty$ as $\alpha = 0$, and this is expected because when $\alpha = 0$ the identifiability conditions are violated.

4. Computer Simulated Performance

The performance of $\hat{\gamma}_1$, $\hat{\gamma}_2$ and $\hat{Q}$ for the case where all three parameters are unknown is shown in Figures 2, 3, and 4.

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F. ASYMPTOTIC PROBABILITY OF ERROR USING TWO DECISION DIRECTED ESTIMATORS FOR TWO UNKNOWN MEAN VECTORS

E. A. Patrick

J. P. Costello

1. Introduction

The asymptotic probability of error for an unsupervised learning system with two unknown mean vectors, which are estimated by "decision-directed" estimators, is investigated. Theoretical solutions, obtained utilizing a digital computer, are obtained for various source probabilities and signal/noise ratios.

This problem is concerned with the difference between the asymptotic probability of error of a particular ("decision-directed") suboptimum learning system and that of the corresponding system with all parameters precisely known. A block diagram of the problem is shown in Figure 1. One of two sources ω_i, i = 1, 2, is active to cause the ℓ-dimensional vector X_s at the s^{th} observation; however, the active source is unknown. The distribution function of X_s, given source ω_i is active, is $F(X_s|\omega_i, \gamma_i)$, where $\gamma_i = E[X_s|\omega_i]$. It is assumed that the source probabilities $P(\omega_i) = Q_i$, i = 1, 2, are known, and we denote $P(\omega_1)$ by Q for convenience. Further, $F(X_s|\omega_i, \gamma_i)$ is assumed gaussian with covariance matrix $\Phi_{xx_i} = \Phi_{xx}$ known.

The problem is, given $X_1, X_2, \ldots, X_{n-1}$, decide which source caused X_n. It is not known which source caused $X_1, \ldots, X_{n-1}$, and γ_1 and γ_2 are unknown. The solution minimizing conditional probability of error computes $f(\gamma_1, \gamma_2|X_1, \ldots, X_{n-1})$.[1] The suboptimum system we are considering substitutes "decision-directed" estimators for γ_1 and γ_2 in place of γ_1 and γ_2, in the optimum multivariate gaussian decision equation which assumes all parameters known.

2. Decision-Directed Estimators for Two Unknown Mean Vectors

The optimum decision equation (with γ_1 and γ_2 known) for deciding which source caused $\mathbf{X}_n$ is

$$(\gamma_1 - \gamma_2)^t \Phi_{xx}^{-1} (\mathbf{X}_n - \mathbf{X}_o) > 0 : \omega_1 \tag{2.1}$$

where

$$\mathbf{X}_o = \frac{\Phi_{xx}(\gamma_1 - \gamma_2)}{(\gamma_1-\gamma_2)^t(\gamma_1-\gamma_2)} \ln \frac{(1-Q)}{Q} + \frac{\gamma_1 + \gamma_2}{2} .$$

A <u>decision-directed estimator</u> $\hat{\gamma}_{i_n}$ for γ_i is defined as a sample mean where samples are used that have been classified as from source ω_i. The algorithm for classification of $\mathbf{X}_n$ is the decision equation rearranged using $\hat{\gamma}_{i_{n-1}}$ in place of γ_i, i = 1, 2. Thus the decision-directed directed estimators are:

$$\hat{\gamma}_{1_n} = \frac{1}{N_1} \sum_{i=1}^{N_1} \mathbf{X}_{n_i} \Big| [-(\hat{\gamma}_{1_{n_{i-1}}} - \hat{\gamma}_{2_{n_{i-1}}})^t \Phi_{xx}^{-1} (\hat{\gamma}_{1_{n_{i-1}}} + \hat{\gamma}_{2_{n_{i-1}}} - 2\mathbf{X}_{n_i}) > 2 \ln (\tfrac{1-Q}{Q})] \tag{2.2}$$

$$\gamma_{2_n} = \frac{1}{n-N_1} \sum_{i=N_1+1}^{n} \mathbf{X}_{n_i} \Big| [-(\hat{\gamma}_{n_{i-1}} - \hat{\gamma}_{2_{n_{i-1}}})^t \Phi_{xx}^{-1} (\hat{\gamma}_{1_{n_{i-1}}} + \hat{\gamma}_{2_{n_{i-1}}} - 2\mathbf{X}_{n_i}) < 2 \ln (\tfrac{1-Q}{Q})] \tag{2.3}$$

since the sequence $\{\mathbf{X}_{n_i}\}_{n_i=1}^{n}$ has a subsequence $\{\mathbf{X}_{n_i}\}$ of size N_1 and a subsequence $\{\mathbf{X}_{n_i}\}$ of size $(n - N_1)$.

Now assuming convergence of the estimators (see 2.), then

$$\lim_{n \to \infty} \hat{\gamma}_{1_n} = E[\mathbf{X}_{n_i} | \tau_\ell] \triangleq H_1 \tag{2.4}$$

$$\lim_{n \to \infty} \hat{\gamma}_{2_n} = E[\mathbf{X}_{n_i} | \tau_u] \triangleq H_2 \tag{2.5}$$

where τ_ℓ means lower truncation at the ultimate decision boundary such that samples to one side of the boundary are classified from source ω_1; and τ_u

means upper truncation such that samples to the other side of the boundary are classified from source ω_2. The distribution function of $\mathbf{X}_n$ is a mixture[1] of two parameter conditional c.d.f.'s so equations (2.4) and (2.5) may be expressed as

$$H_1 = E[\mathbf{X}_n|\omega_1,\tau_\ell]\ P[\omega_1|(D_n > \delta,\tau_\ell)] + E[\mathbf{X}_n|\omega_2,\tau_\ell]\ P[\omega_2|(D_n > \delta,\tau_\ell)]$$

$$H_2 = E[\mathbf{X}_n|\omega_1,\tau_u]\ P[\omega_1|(D_n < \delta,\tau_u)] + E[\mathbf{X}_n|\omega_2,\tau_u]\ P[\omega_2|(D_n < \delta,\tau_u)]$$

where

$$D_n = -(H_1 - H_2)^t\ \Phi_{xx}^{-1}\ (H_1 + H_2 - 2\mathbf{X}_n),\ \delta = 2\ \ln\ (\frac{1-Q}{Q}). \tag{2.6}$$

3. Asymptotic Probability of Error

Defining $s_1 = [H_1 - H_2]^t\ \Phi_{xx}^{-1}\ \mathbf{X}$, then s_1 is perpendicular to the decision hyperplane. A basis can be constructed so that the other components of $\mathbf{X}$, $\{s_i\}_{i=2}^{\ell}$ are parallel to the hyperplane and uncorrelated with s_1 so the probability of error analysis can be reduced to one dimension.

Define

$$\beta_i = [H_1 - H_2]^t\Phi_{xx}^{-1}\ H_i\quad ,\quad m_{i_1} = \frac{(\gamma_1 - \gamma_2)^t\Phi_{xx}^{-1}\ \gamma_i}{[(\gamma_1-\gamma_2)^t\Phi_{xx}^{-1}\ (\gamma_1 - \gamma_2)]^{\frac{1}{2}}}\ (\beta_1-\beta_2)^{\frac{1}{2}} \tag{3.1}$$

and

$$= [H_1 - H_2]^t\ \Phi_{xx}^{-1} \tag{3.2}$$

The variance and the decision boundary are then

$$E[s_1 s_1{}^t] = \beta_1 - \beta_2 \tag{3.3}$$

$$s_o = \ln\ (\frac{1-Q}{Q}) + \frac{\beta_1 + \beta_2}{2} \tag{3.4}$$

Since s_1 is truncated, from Cramer[7] its mean is

$$E[s_{1_n}|\omega_i,\tau] = m_{i_1} + \lambda\sigma \tag{3.5}$$

where

$$\lambda = \lambda_{\ell}(r_i) = \frac{\phi'(r_i)}{1-\phi(r_i)} \qquad \text{lower truncation}$$

$$\lambda = \lambda_u(r_i) = -\frac{\phi'(r_i)}{\phi(r_i)} \qquad \text{upper truncation}$$

$$r_i = \frac{s_o - m_{i_1}}{\sigma} = \frac{2\ln(\frac{1-Q}{Q}) + (\beta_1 + \beta_2 - 2m_{i_1})}{2(\beta_1-\beta_2)^{\frac{1}{2}}} \tag{3.7}$$

and $\phi(X)$ is gaussian mean zero, variance of one.

Applying (3.2) to (2.6) and substituting the term definitions,

$$\beta_1 = \frac{Q_1(1-\phi(r_1))}{D_1}\{(m_{1_1} - m_{2_1}) + (\lambda_{\ell}(r_1) - \lambda_{\ell}(r_2))\sigma\} + \{m_{2_1} + \lambda_{\ell}(r_2)\sigma\}$$

$$\beta_2 = \frac{Q_1\phi(r_1)}{D_2}\{(m_{1_1} - m_{2_1}) + (\lambda_u(r_2) - \lambda_u(r_2))\sigma\} + \{m_{2_1} + \lambda_u(r_2)\sigma\} \tag{3.8}$$

where

$$D_1 = Q_1(1 - \phi(r_1)) + Q_2(1 - \phi(r_2)), \quad D_2 = Q_1\phi(r_1) + Q_2\phi(r_2)$$

Hence equations (3.8) for β_1 and β_2 can be solved numerically on a digital computer as a function of the source probabilities Q_1, and the SNR of the received samples X_n.

If m_{1_1} and m_{2_1} were known, the probability of error would be

$$(P_e)_{min} = Q_1\,\phi(a_1) + (1 - Q_1)(1 - \phi(a_2)) \tag{3.9}$$

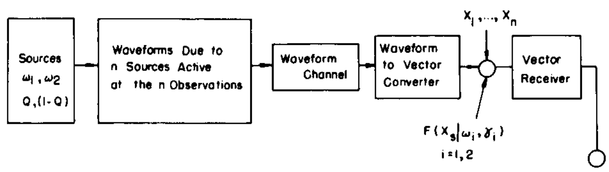


Figure 1 Problem Model

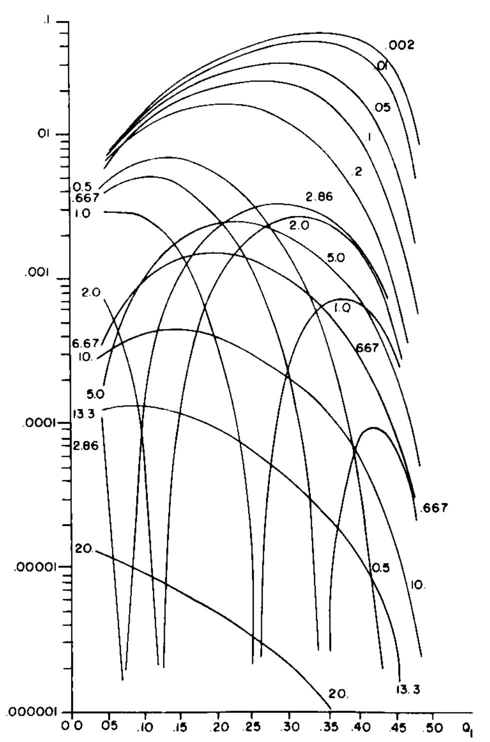
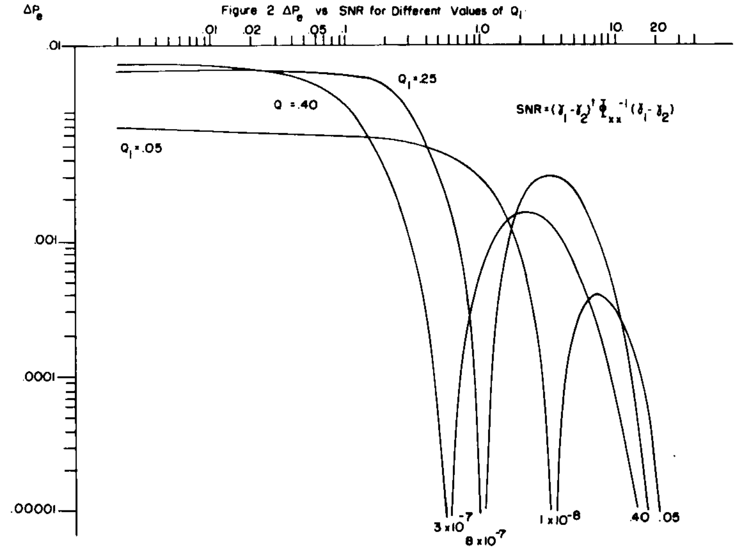


Fig 3 ΔP_e vs Q_1 for Different Values of $SNR = (\gamma_1 - \gamma_2)^T \Phi_{xx}^{-1} (\gamma_1 - \gamma_2)$

Figure 2 ΔP_e vs SNR for Different Values of Q_1



where

$$a_1 = \frac{\delta - (m_{1_1} - m_{2_1})}{2(m_{1_1} - m_{2_1})^{\frac{1}{2}}} \qquad a_2 = \frac{\delta + (m_{1_1} - m_{2_1})}{2(m_{1_1} - m_{2_1})^{\frac{1}{2}}} \tag{3.10}$$

$$m_{1_i} = (\gamma_1 - \gamma_2)^t \, \Phi_{xx}^{-1} \gamma_i$$

where $(P_e)_{min}$ indicates it is the minimum probability of error. When m_{1_1} and m_{2_1} are unknown they are estimated by β_1 and β_2 respectively such that the probability of error is

$$(P_e)_{sub} = Q_1\, \emptyset(r_1) + (1 - Q_1)(1 - \emptyset(r_2)) \tag{3.11}$$

where $(P_e)_{sub}$ indicates a suboptimum probability of error. The difference in the probabilities of error was defined ΔP_e, and sets of curves are presented in Figures 2 and 3 showing ΔP_e vs SNR with Q_1 as a parameter and ΔP_e vs Q_1 with SNR as a parameter respectively.

5. Conclusions

Decision-directed estimators were used in place of the fixed but unknown mean vectors in a binary, multivariate gaussian decision equation. Assuming these estimators converge to constants (Eq.'s 2.4 and 2.5), the asymptotic probability of error for the resulting suboptimum system was evaluated. The difference ΔP_e in asymptotic probability of error between the optimum system with γ_1 and γ_2 known and the suboptimum learning system is plotted in Fig.'s 2 and 3. Figure 3 shows that the asymptotic performance of the suboptimum system is not much poorer than the system with all the parameters known. For example, $\Delta P_e < 10^{-2}$ for all Q_i if SNR $\tilde{>}$ 0.5 and $\Delta P_e < 10^{-3}$ for SNR $\tilde{>}$ 10. For the special cases where the sources are equally likely or one source is active, the asymptotic performance of the suboptimum system is the same as the known parameter system.

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II. SIGNAL DESIGN

A. M-ARY SIGNAL SETS FOR SCATTER-MULTIPATH AND OTHER ASYNCHRONOUS COMMUNICATION*

D. R. Anderson

As indicated by Anderson[1] the optimum equal-energy, common bandwidth occupancy binary signal set for the Turin and Price scatter-multipath channel models is characterized by the following property: the peak of the envelope of the cross-correlation is essentially a constant multiple of $(2TB)^{-\frac{1}{2}}$, TB being the common time-bandwidth product. This condition also characterizes the optimal binary signal set with the same constraints for non-synchronous, phase-incoherent reception in the presence of additive white noise. In the case of M-ary signal sets, the condition for optimality when there is common frequency occupancy is not known. However, both Lerner[2] and Yates and Cooper[3] have found constant-envelope M-ary signal sets with common frequency occupancy and common TB-product equal to M for which the peak of the envelope of the cross-correlation of every pair is at most $\ln M/M^{\frac{1}{2}}$. In view of the condition for optimality in the binary signal-set case, these results appear to be close to optimal. The current paper exhibits a method of using shift register sequences[4] to obtain constant envelope p-ary signal sets for every prime p and every $n \geq 1$. These signal sets have the property that:

1) every signal set is generatable by a p-ary shift register.
2) all the members of a given set have a TB product equal to p.
3) for a given p^n-ary signal set, all envelopes of crosscorrelation functions are at most $\ln(p^n)/p^{\frac{1}{2}}$.

*This research supported by NASA Grant NsG-553.

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B. ADAPTIVE MODULATION OF SIGNALS FOR A RAYLEIGH FADING MEDIUM

J. F. Hayes

Optimum modulation criteria have been derived for binary signals transmitted over Rayleigh fading multipath channels disturbed by stationary additive Gaussian noise. The basic assumption is that the receiver has learned the gain, phase shift and delay in each multipath and the transmitter has learned the gain in each multipath. The transmitter uses its knowledge to modulate the amplitude of each transmitted pulse so as to minimize the average risk subject to a constraint on the average transmitted energy.

Let the transmitted signal be $\pm A(X)\phi(t)$ where $X \triangleq \sum_{i=1}^{n} a_i^2$, the sum of the squares of the gain in each multipath. Probability of error is minimized under an average transmitted energy constraint when $A(X)$ satisfies the following implicit equation

$$A(X) = \lambda X^{\frac{1}{2}} \exp[-2A(X)X]$$

$X \geq 0$ where λ is a Lagrange multiplier. This solution is intuitively satisfying since for X small $A(X)$ is small. Also for X large the transmitted energy is small while the received energy is large.

The use of this modulation scheme at the transmitter yields an improvement in performance which increases with receiver SNR. For low SNR (less than 0db) the improvement is small. However, for moderate SNR it can be significant. For example, for probability of error equal to 10^{-4} and four multipaths there is a savings of approximately 4 db.

The transmitter may obtain its knowledge of the channel by a noiseless feedback link between the receiver and the transmitter. In the case of two-way transmission where the multipath channel is the same in both directions, the noiseless feedback channel is not necessary since the channel parameters learned at each receiver may be used to modify transmission. It is assumed in both cases that the channel is slowly varying so that the transmitter's knowledge is up-to-date.

The case where binary on-off signals are transmitted has also been examined. The results are similar to those quoted above.

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III. ADDITIONAL COMMUNICATION SCIENCES RESEARCH AREAS

A. CORRELATION DETECTOR*

P. A. Wintz

R. A. Markley

It is well known that the optimum detector of known signals in white gaussian noise is a correlation detector. The correlation detector must correlate the received data with replicas of the known transmitted signals, and decide in favor of the signal having the largest correlation.

The design and testing of a binary detector using Hall effect multipliers has been started. Circuitry for generating and synchronized sample and dump pulses, an analog integrator and dump circuitry, and a sampler have been designed, and testing is nearly complete. Work now in progress includes testing and adapting the Hall multipliers to the system and design at the decision logic circuitry.

B. PERFORMANCE OF SELF BIT SYNCHRONIZING SYSTEMS**

P. A. Wintz

E. J. Luecke

In digital systems it is desirable to obtain the bit synchronizing signals that are necessary for correlation detectors directly from the information symbol sequence. For binary, antipodal signaling, the discrete components of the power spectrum are given by[1]

$$|(2P-1)\ G(j2\pi f)|^2 \sum_{N=-\infty}^{+\infty} \delta(f-nf_B) \qquad (1)$$

where G(s) is the Laplace transform of the symbol waveform, P is the probability of a positive symbol, and f_B is the symbol rate. Since bit synchronizing

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signals can be obtained only if there is finite power at f_B, nonlinear filtering is necessary if reliable operation is to be maintained as the symbol probabilities approach .5.

Maximum likelihood estimation techniques yield an optimum nonlinear operation for obtaining synchronization from a finite length record of exactly N symbol duration of received signal. If X is the received signal samples vector and ϵ is the synch position $(0 < \epsilon < 1/f_B)$ then $p(\epsilon/X)$ is given by

$$\ln \cosh (X_0^T \Phi_{NN}^{-1} S(\epsilon) + \sum_{j=1}^{N-1} \ln \cosh (X_j^T \Phi_{NN}^{-1} S(0) + \ln \cosh (X_N^T \Phi_{NN}^{-1} S(1-\epsilon) \quad (2)$$

where $S(y)$ is the vector of symbol samples shifted by y and X is partitioned so that $X = (X_0, X_1 \ldots\ldots X_N)$ with X_0 being of length ϵ, $X_1 \ldots\ldots X_{N-1}$ length $1/f_B$, and X_N length $(1-\epsilon)$. Monte Carlo simulation for half sine and square symbols yields data from which the degradation of probability of error for a correlation detector which is synchronized by this method is computed.

A sub-optimum solution to the problem is shown below.

Signal and Additive Noise	→	Lowpass Filter	→	Full Wave Squared Law Device	→	Bandpass Filter Centered at f_B	→	Infinite Clipper, Differentiator, and Half Wave Rectifier	→

The output noise, in the form of synchronizing pulse jitter, is a result of both input noise and of lowpass filtering of the signal random process. The output of the bandpass is approximated by a sine wave plus Gaussian noise and the signal and noise powers at the output are computed by standard techniques.[2,3] From this the probability density of synchronizing error is computed. For various S/N and square, half sine, and raised cosine symbol waveshapes, optimum lowpass cutoff is determined and detection error is computed. These results are compared with the maximum likelihood synchronizer.

It is shown that half sine or raised cosine symbols produce better synchronizer performance and also better detector performance than does the square symbol.

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C. DIGITAL SIMULATION OF A RANDOMLY TIME-VARIANT COMMUNICATION CHANNEL*

J. C. Lindenlaub

C. C. Bailey

The program in simulation of communication systems[1] is being extended to the simulation of systems employing randomly time-variant channels. The first phase of this program is the development of digital computer programs to simulate the effects of randomly time-variant channels. A set of programs is now available which can provide simulation of a general class of channels. This simulation system has been applied to the case of tropospheric scatter communication channel.

The basic theory for the simulation system is due to Stein.[2] This requires that the channel model be simulated to obey the following assumptions:

1. The channel is a linear time-variant system. Thus it has a time-varying impulse response $h(\xi,t)$ and an associated time-varying transfer function $H(f,t) = F_{\xi}\{h(\xi,t)\}$

2. The channel impulse response, $h(\xi,t)$, is stationary complex Gaussian random process.

*Supported by NASA Grant NsG-553.

3. Only bandlimited signals are used as inputs to the channel.
4. The correlation function of the channels' time-varying transfer function, $R_H(\Omega,\delta) = E[H(f,t)H^*(f + \Omega,t + \delta)]$ is factorable into a function of Ω and a function of δ, i.e. $R_H(\Omega,\delta) = v(\Omega)q(\delta)$

The tropospheric scatter channel model used for the testing of the simulation system is the Sunde model.[3] For this model, the channel correlations are

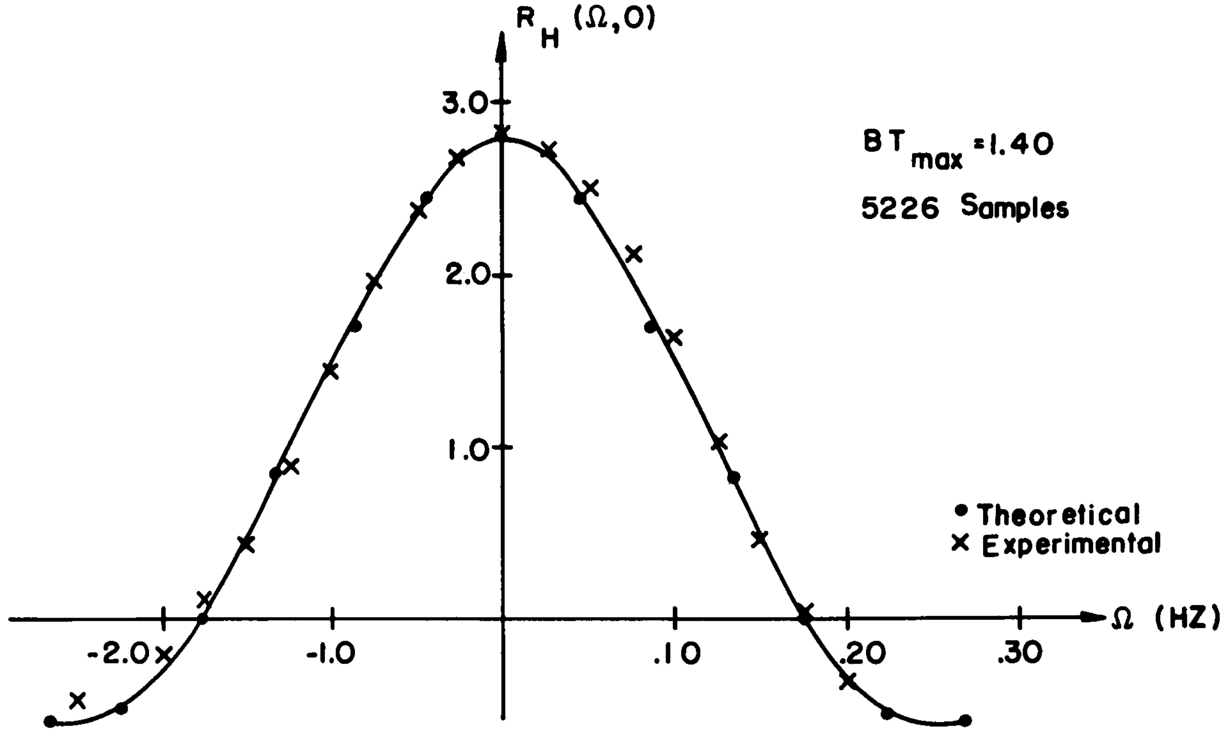
$$v(\Omega) = 2R_o \frac{\sin 2\pi\Omega T_{max}}{2\pi\Omega T_{max}}$$

$$q(\delta) = \exp\left[-\frac{\sigma^2\delta^2}{2}\right]$$

where T_{max} is the maximum departure from the average transmission delay over the channel, R_o is the mean square value of the channel gain, and σ is a measure of the rapidity with which channel fluctuations take place. This channel model has been simulated for the following parameter values

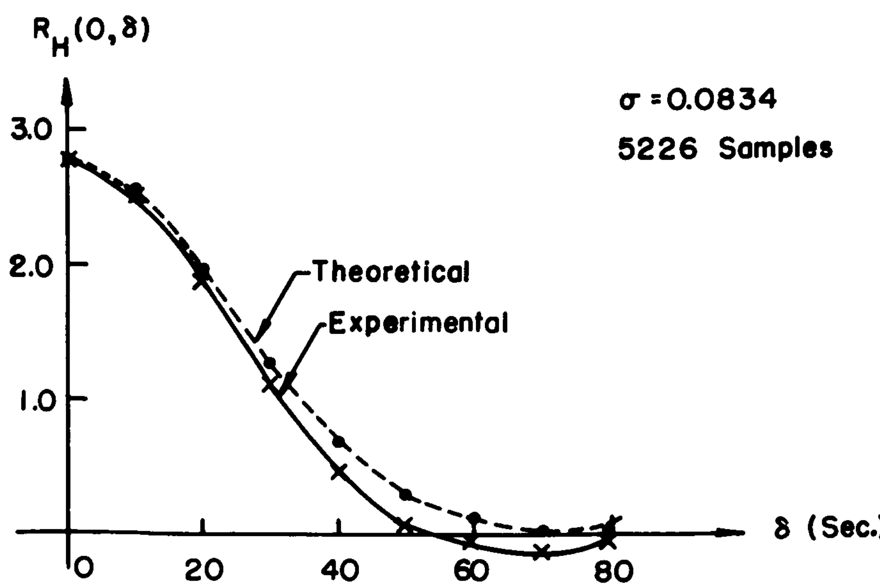
1) input signal bandwidth = $\frac{1}{2}$ Hz.
2) T_{max} = 2.8 sec.
3) $R_o = 2$
4) σ = .0834 sec^{-1}

Experimental measurements of $v(\Omega)$ and $q(\delta)$ have been made with this simulation system. The measurement of $q(\delta)$ is performed by exciting the channel with a sinusoid, demodulating the channel output with a synchronous detector, and autocorrelating the output of the detector. The measurement of $v(\Omega)$ is performed by exciting the channel with two sinusoids whose frequencies differ by Ω Hz, demodulating the channels' response to each sinusoid with synchronous detectors, and crosscorrelating the outputs of the two detectors. Figures 1



Frequency Correlation Function $R_H(\Omega,0)$ vs. Ω

Figure 1



Time Correlation Function $R_H(0,\delta)$ vs. δ

Figure 2

2 show theoretical and experimental graphs of $v(\Omega)$ and $q(\delta)$ for this system.

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D. SIGNAL DEBLURRING*

C. D. McGillem

S.S.C. Yao

When a signal is passed through a system, the signal is frequently altered as a result of interaction with the system. This process may be thought of as "blurring" of the signal. In many instances the exact nature of the blurring is known, and the question is raised as to the possibility of recovering the original signal by suitable processing of the blurred signal. Some examples will illustrate the nature of the problem. Consider the recording of an electrical signal on a magnetic tape. Due to the finite width of the airgap in the recording head, the signal is simultaneously applied to a finite length interval on the tape. Accordingly then, the intensity of recording at any point on the tape is the integrated value of the signal that occurred while the tape passed under the airgap, and is thus a "running average" of the actual instantaneous signal. Mathematically the recorded signal can be expressed in terms of the actual signal as

$$y(t) = \int_{t-T/2}^{t+T/2} x(\xi)\, d\xi \tag{1}$$

*This work supported by NASA Grant NsG-553.

where $x(t)$ is the original signal, $y(t)$ is the recorded signal, and T is the time required for a point to move through the airgap. It can be readily shown that the integral above is equivalent to convolving the signal with a rectangular pulse of unit height, thus

$$y(t) = x(t) * h(t) \tag{2}$$

where $h(t)$ is the blurring function--in this case a rectangular pulse of width T centered at the origin. Ideally, one would solve a problem of this sort by using the Fourier or Laplace transform. Formally this leads to

$$Y(\omega) = X(\omega)\, H(\omega)$$

$$X(\omega) = Y(\omega) \cdot \frac{1}{H(\omega)}$$

$$x(t) = y(t) * F^{-1} \left\{\frac{1}{H(\omega)}\right\} \tag{3}$$

Unfortunately $F^{-1} \left\{\frac{1}{H(\omega)}\right\}$ does not in general exist for functions, $h(t)$, of interest. In the present case

$$H(\omega) = F\{h(t)\} = T \frac{\sin \omega T/_2}{\omega T/_2} \tag{4}$$

$$\frac{1}{H(\omega)} = \frac{1}{T} \frac{\omega T/_2}{\sin \omega T/_2} \tag{5}$$

This is not a well-behaved function since it is unbounded as $\omega \to \infty$, and furthermore has infinite discontinuities at $\omega = \frac{2n\pi}{T}$, $n = 1,2,\ldots$ There is, accordingly, no inverse transform. There are, nevertheless, ways to handle problems of this sort, and it is the purpose of this research to investigate a number of these methods. Before discussing the methods, some additional examples where deblurring would be important should be mentioned. Photographs that are blurred due to image motion or defocussing are an example of process where the blurring operation is known precisely, and recovery of the original would be highly important. In such cases the deblurring is two dimensional. Transmission of waveforms through channels of limited bandwidth causes distortion that can be removed by deblurring techniques. Beam sharpening in radar and crisping by T.V. pictures are other examples of deblurring.

Ultimately the limitation on deblurring will be determined by noise present in the system or in the processing itself. However, much improvement is possible even with noise present.

Several methods of deblurring are being investigated. In one method the reciprocal of the transform of the blurring function is approximated by various infinite series. The inverse transformation is then carried out on a term-by-term basis, and deblurring is obtained by convolution of the blurred signals with partial sums of these terms. Is has been demonstrated that such procedures are valid in many cases of interest, but generally appear to be limited to signals with only modest amounts of blurring. In the presence of heavy blurring the partial sums are divergent.

Another method that is being studied is to solve the integral equation (2) for particular blurring functions (kernels) by working in the time domain. For example, solutions for rectangular pulses, single loop cosine and raised cosine pulses, and Gaussian pulses have been obtained. Such solutions involve derivatives of the blurred signal. In the case of the rectangular pulse, only the first derivative is required to obtain an exact recovery of the signal, whereas for the Gaussian pulse derivatives of all orders are required. This method of approach appears very promising, and further studies are being made.

E. THEORY OF RANDOM PROCESS

J. A. McFadden

James L. Lewis, III

In a previous report,[1] a stationary Gaussian process Y(t) was defined as <u>locally Markov</u> if and only if its autocorrelation function was asymptotically linear near the origin, with negative slope. Thus, if the mean value $E[Y(t)]$ is zero, then the

$$\rho(\tau) \equiv \frac{E[Y(t)\ Y(t+\tau)]}{E[Y^2(t)]} = 1 - b\ |\tau| + O(\tau^2), \text{ where } b > 0.$$

A paper describing this work was submitted to a journal, and has now been resubmitted under the title, "On a Class of Gaussian Processes for which the Mean Rate of Crossings is Infinite," by J. A. McFadden.

Another paper, "Multivariate Normal Integrals for Highly Correlated Samples from a Wiener Process," by J. A. McFadden and James L. Lewis, has now been greatly augmented, revised, and resubmitted to a journal.

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F. PHASE LOCK LOOP STUDIES*

J. C. Lindenlaub

J. J. Uhran

The study to determine the effects of the class of $n^{\underline{th}}$ order tanlock phase detector characteristics upon the design parameters of first and second order phase lock loops has been completed. During the past six months the first order system results, which were reported on in the last semi-annual report, have been augmented with second order system results, and a technical report summarizing the work has been prepared (TR-EE66-19).

The main findings of the study are that the lock range capabilities of the $n^{\underline{th}}$ order tanlock systems exceed those of the sine comparator for high signal to noise ratios, but fall off faster than the sine comparator as threshold is approached. Synchronization times of the $n^{\underline{th}}$ order tanlock systems are superior

*Supported in part by NASA Grant NsG-553.

to the ordinary phase lock loop. When normalized to equivalent noise bandwidth, the tanlock systems exhibit a higher threshold, but when normalized to lock range, the tanlock systems have a slightly lower threshold. We have defined threshold in terms of the rate of cycle slipping. Curves showing these characteristics may be used to aid in the design of tanlock or ordinary phase lock loop systems.

G. A PHASE LOCK LOOP SYSTEM WITH A MODULO $2n\pi$ PHASE DETECTOR*

J. C. Lindenlaub

D. P. Olsen

As a continuation of the study to determine the effects of phase detector characteristics on phase lock loop design parameters discussed above, a study has been initiated on a phase lock system with a phase detector characteristic that is linear and periodic with period $2n\pi$ radians instead of the usual 2π radians. Of particular interest is the threshold behaviour of the modulo $2n\pi$ system. The mathematical or experimental analysis of the threshold performance of such a system has not been noted in the literature.

It is felt intuitively that such a phase lock loop has a lower threshold and intermodulation distortion than the conventional phase lock loop. It is believed that the larger phase error range will decrease the probability of cycle slipping. This will decrease the noise due to 2π impulses in the output and hence decrease the threshold. In addition, the linearity inherent in this phase detector will decrease the intermodulation distortion at the output.

A phase lock loop using this type phase detector ($n = 2^i$, i=1 to 9) is currently under construction. Figure 1 is a block diagram of the system. In area 1 of this figure the input signal, consisting of a frequency modulated

*Supported by NASA Grant NsG-553.

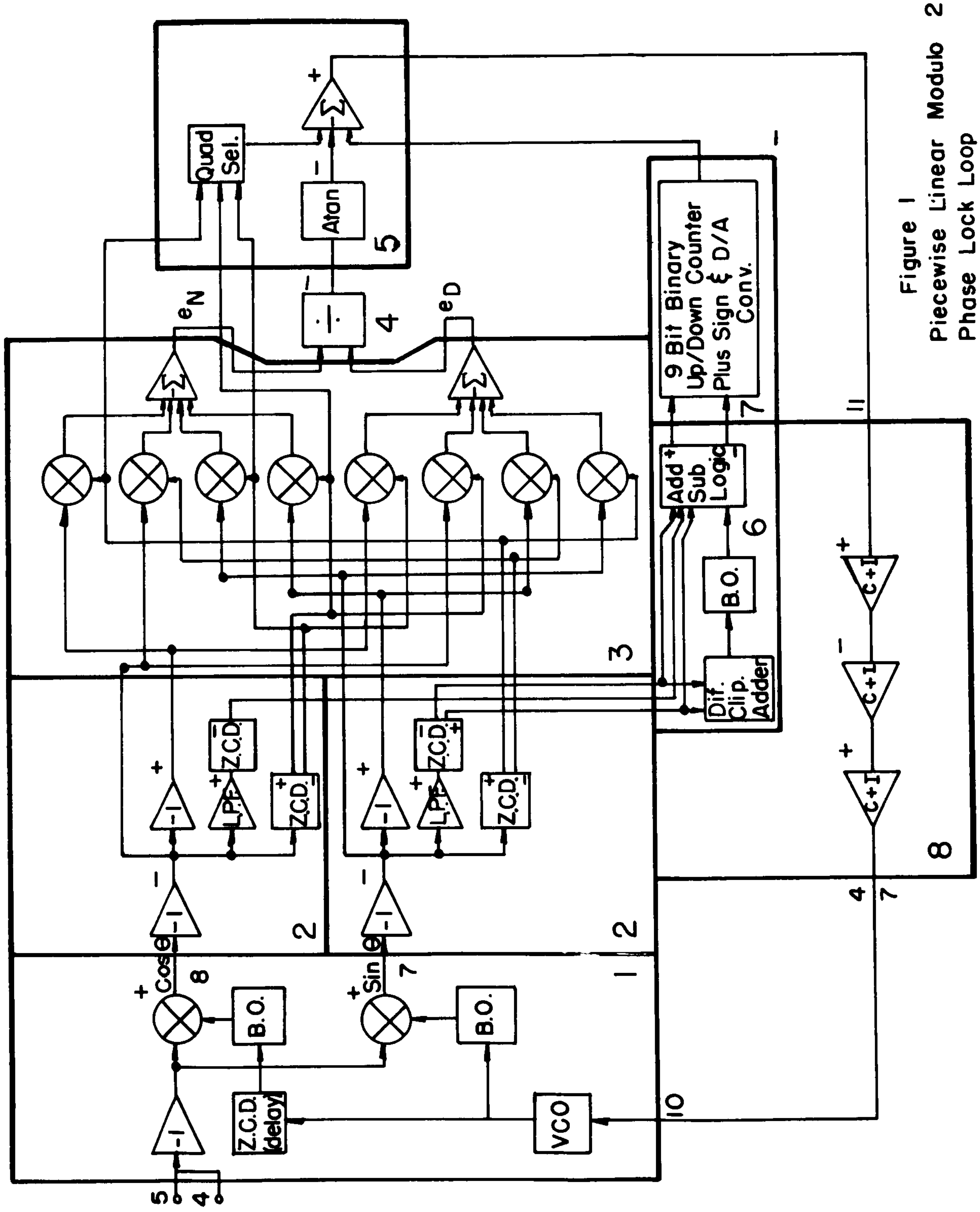

Figure 1
Piecewise Linear Modulo 2nπ
Phase Lock Loop

carrier plus band limited noise, is sampled by a locked reference oscillator. This is done by two sample and hold circuits operating in quadrature. The yield is two signals proportional to the sine and cosine of the phase error between the input signal and the reference oscillator. In area 2 the sign of the sine and cosine voltages is detected and used to control a gating network that feeds two adders in area 3. The output of the upper adder is

$$e_n = A[\text{SIGN}(\sin\theta_e)\cos\theta_e - \text{SIGN}(\cos\theta_e)\sin\theta_e]$$

and for the lower adder is

$$e_d = A[\text{SIGN}(\cos\theta_e)\cos\theta_e + \text{SIGN}(\sin\theta_e)\sin\theta_e]$$

$$= A[|\cos\theta_e| + |\sin\theta_e|]$$

where $A > 0$ is the input signal amplitude. The output of the adders enter area 4 where an analog divider calculates

$$\frac{e_n}{e_d} = -\tan\left[\frac{\pi}{4} + \theta_e - (K-1)\frac{\pi}{2}\right]$$

where K is the quadrant of θ_e modulo 2π. It can be shown that

$$A/\sqrt{2} < e_d < A\sqrt{2}.$$

Thus the analog divider denominator is never zero and always positive for $A > 0$. Therefore the divider never saturates; also the divider output is independent of A.

In area 5 an inverse tangent operation upon the output of the analog divider produces the phase, θ_e, modulator $\pi/2$. The nonlinear inverse tangent operation is approximated to within $\pm$ 1% by 7 line segments over the $\pm\,\pi/4$ range. A signal determined by the quadrant of the modulo 2π phase is added to the modulo $\pi/2$ phase to produce the modulo 2π phase. The quadrant is determined by a logic circuit connected to the sign ($A\sin\theta_e$) and sign ($A\cos\theta_e$) detectors in area 2. Also a signal proportional to the integral number of cycles slipped is added to the modulo 2π phase to obtain the modulo $2n\pi$ phase.

In area 2 the A sin θ_e and A cos θ_e signals are individually operated upon by single pole low pass filters to improve the signal to noise ratio. Then the sign of each filtered signal is determined.

In area 5 a logic circuit operating on the sign of the filtered sine and cosine is used to determine the occurrence and direction of cycle slipping. That is, if $F(\sin \theta_e)$ becomes positive while $F(\cos \theta_e)$ remained negative, a negative cycle slip has occurred. (Here $F(\cdot)$ denotes the filtered signal). Similarly if $F(\sin \theta_e)$ becomes negative while $F(\cos \theta_e)$ remains negative, a positive cycle slip has occurred.

Area 7 is a 9 bit up-down counter plus sign bit and digital to analog converter. If a positive cycle slip occurs, one is added to the count stored in the counter. Similarly, if a negative cycle slip occurs, one is subtracted. The analog output signal is proportional to the count including its sign. This signal is added to the modulo 2π phase in area 5 as mentioned above.

Area 8 is the loop filter in which any combination of poles and zeros up to 3 each can be realized through use of 3 operational amplifiers connected as ideal integrators. The output of the filters is used in area 1 to control the frequency of the reference oscillators.

The prototype models of areas 2,3,5, and 8 have been constructed in the lab, and work individually. Area 4 will be a Philbrick analog divider. The sample and hold circuits in area 1 are presently under development.

H. AN ANALYSIS OF HIGH ORDER PHASE-LOCKED LOOP BEHAVIOR IN THE PRESENCE OF WHITE NOISE

D. R. Anderson

J. Y. S. Luh

The analysis of the nonlinear phase-locked loop behavior in the presence of white noise was attempted by various authors. Among them Tikhonov [1,2] and Viterbi [3] introduced a technique based on the solution of the Fokker-Planck equation. Their attempts were successful for the first order loop in which the probability distribution of the phase-error is of modulo 2π. The modulo 2π representation, however, allows the phonomenon of slipping cycles which is not desirable. Viterbi [3], and Charles and Lindsey [4] improved the results by analyzing approximate models of the second order loop. No results on the third or higher order loops are known.

This analysis introduces a method of investigating the higher order phase-locked loop. In practice the high order loop becomes more important if the received signal phase is of the form of $\alpha t^2 + \omega t + \theta$, or if the data processing of the received signal is involved. The scheme is based on the solution of the backward diffusion equation, and the analysis is an application of the results of Mishchenkoś pursuit problem [5].

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I. RANDOM SIGNAL RADAR*

G. R. Cooper   W. B. Waltman

C. D. McGillem   R. A. Emmert

An experimental random signal radar system has been assembled and is currently undergoing tests. A block diagram of the experimental system is shown in Fig. 1 and a photograph of the assembled equipment is shown in Fig. 2. The parameters of the system as it is presently constituted are as follows:

Frequency	8.995 GHz
Transmitted Power	500 μw
Bandwidth	120 MHz
Receiver Noise Figure	9 DB
Antenna Beamwidth	5°
Range Resolution	≈ 2m
Delay	500 μsec

Although the correlator is basically a digital device, the output has been converted to an analog signal to speed up operation. As presently arranged the sampling gates are moved relative to each other, providing a linearly varying delay. By suitable modulation of the local oscillator, the phase of the correlation function is caused to vary periodically, thereby giving a sinusoidally modulated output of the correlator at each delay. This output is passed through a bandpass filter and then into an envelope detector. The output of the detector is proportional to the envelope of the correlation function at any delay. By connecting one axis of an X-Y recorder to a voltage proportional to the delay between the sampling gates, and the other axis to the system output, a plot of correlation function envelope versus delay (range) is

*Supported by NASA Grant SC-NsG-543.

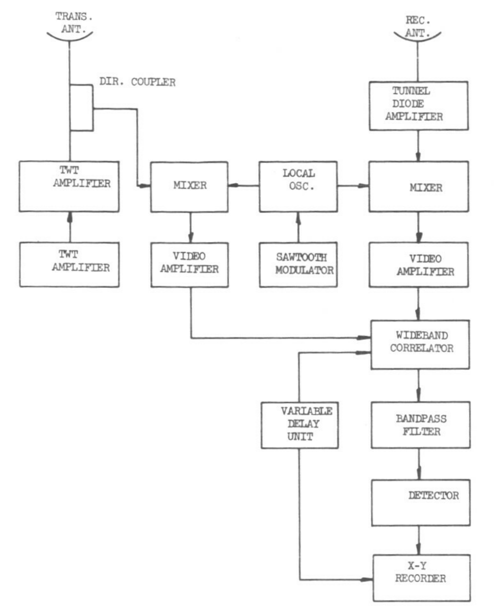
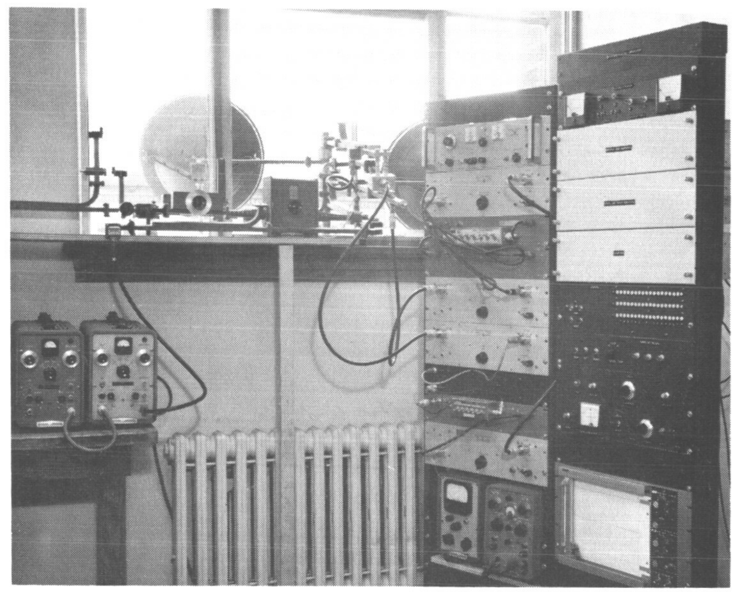


Fig. 1. Random Signal Radar



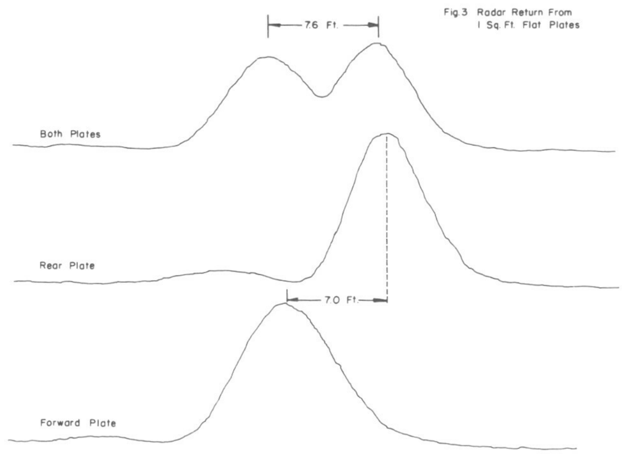


Fig.3 Radar Return From 1 Sq.Ft. Flat Plates

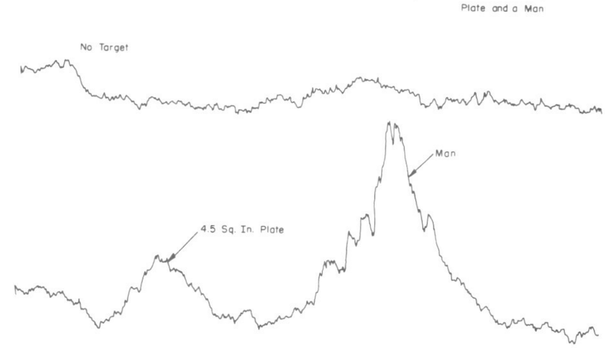


Fig. 4 Return From a Small Plate and a Man

obtained.

Figure 3 shows a series of returns from two one foot square flat plates. The plates were located approximately 150 feet from the transmitting antenna, and only the portion of the return corresponding to the immediate vicinity of the targets is shown. The target spacing was seven feet.

Figure 4 shows the return from a man and a flat metal plate target having a cross-sectional area of 4.5 sq. in. It is evident that the man provides a much larger scattering cross-section. In fact, this technique can be used to make accurate comparisons between the scattering cross-sections of targets. By direct scaling of Fig. 4, it is found that the response to the man is 3 times greater than that of the 4.5 sq. inch target, and therefore the radar cross-section of the man is 9 times the radar cross-section of the small plate.

The various experimental data collected to date have been primarily for the purpose of aiding in the system development or to verify predictions of system performance. The next phase will involve radar analysis of particular types of targets.

J. ESTIMATION OF SONAR TARGET PARAMETERS*

G. R. Cooper

J. U. Kincaid

The parameter estimation technique discussed in the 4th Semi-Annual Research Summary has been found not acceptable due to a singular matrix which cannot be eliminated.

Under consideration at the present time is a procedure which estimates the time of return of the pulses from the target through the multipath channel.

*Supported by NSF Grant GK-189, PRF 4243.

These estimates then produce an arrival time pattern which is used to estimate the most likely multipath structure. This seems possible since the entire multipath pattern is produced for each target reflecting point.

Using this multipath structure estimate it is possible to formulate

$$I = \sum_{i=1}^{N} [x(t_i) - \hat{x}(t_i)]^2 \tag{1}$$

where $x(t_i)$ is the received signal at the sampling time t_i; and

$$\hat{x}(t_i) = \sum_{\ell=1}^{L} \hat{m}_\ell(t_i) \sum_{j=1}^{K} \hat{a}_j \, s[\hat{\beta}_j(t_i - \Delta_\ell) - \tau_{jo}] \tag{2}$$

The parameters L, K, τ_{jo}'s and the Δ_ℓ's are obtained in the multipath structure estimation. The $\hat{a}_j$'s, $\hat{\beta}_j$'s and $\hat{m}_\ell(t_i)$'s are now selected so as to minimize (1).

At the present time consideration is being given to testing the above procedure for a known target, and using a simulated "channel".

SECTION 5.2

NONLINEAR CIRCUITS AND SYSTEMS

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N67-33611

I. FUNDAMENTAL PROBLEMS OF CLASSES OF NETWORKS

A. STATE VARIABLES, ENERGY FUNCTIONS, AND PR MATRICES

B. J. Leon

P. M. Lin

In recent years there has been considerable interest in the relation between the states of a linear, time-invariant circuit and the port variables.[1,2,3,4] From the port viewpoint the passivity of a circuit is completely determined by the hybrid matrices. If at least one such matrix exists and is PR, the system is passive. If the system is not both controllable and observable, there is the obvious possibility that the port specification does not give the whole story. Even when all natural frequencies are evident in the transferfunction matrix, there is more to the internal structure than is evident from the ports.

One concept that has proven useful in passive network theory is the energy function concept.[5] When the circuit is not RLC, the first problem is the definition of energy functions so that power into the circuit is the difference between rate of change of stored energy and the energy dissipated. Once a satisfactory separation of the energy is established, the connection between internal passivity as evidenced by the positive semi-definiteness of the energy functions and external passivity as evidenced by the PRness of the hybrid matrix becomes the basic problem. Clearly internal passivity implies external passivity. The converse is not true.

Preliminary results on this problem were reported at the Allerton Conference.[6] The next question to be answered is to ascertain the region of state space that is reachable from points in the port space. This mapping from port space to state space may hold the clue to the relation between internal passivity and external passivity.

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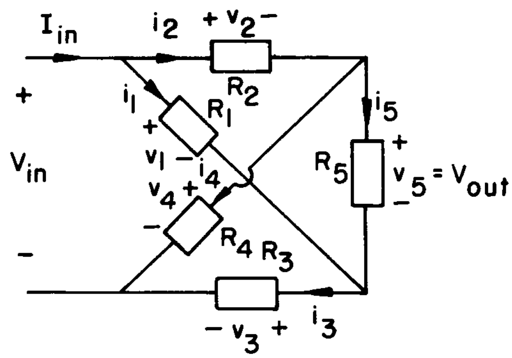
B. COMPUTER ANALYSIS AND SYNTHESIS OF NONLINEAR NETWORKS

W. H. Stellhorn

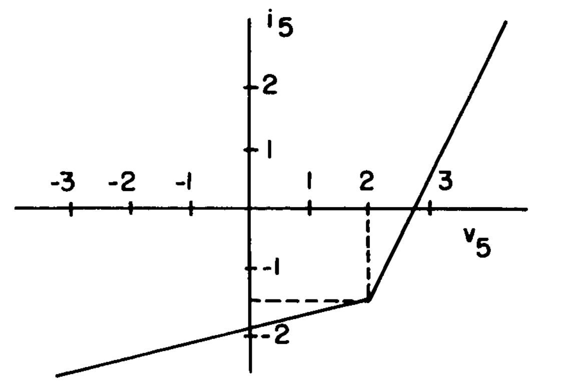
L. O. Chua

A series of computer programs is being developed for iterative piecewise linear analysis and synthesis of nonlinear networks.[1,2] Two versions of the analysis program, one for very general applications and the other for efficient processing of a particular class of nonlinear circuits, are nearly completed. A synthesis program has also been written and partially debugged. When completed, it will accept as specifications the required driving point plot, input voltage to output voltage transfer curve, and load characteristic, and will realize a lattice structure with the desired properties.

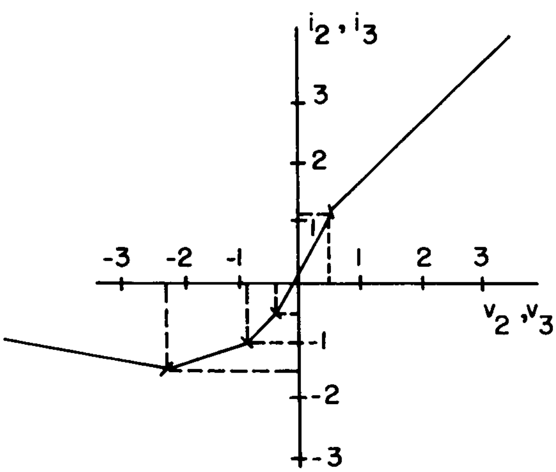
Both analysis programs are capable of determining the driving point characteristics and any of the four possible transfer curves of any memoryless network for which the required equations can be written. Because the user supplies the necessary equations, circuit elements are permitted which would be difficult or impossible to include if the equations were generated by the program; e.g., controlled sources, gyrators, and ideal transformers. The network may contain as many as ten nonlinear resistors, each represented in the v-i plane by between one and ten piecewise linear segments. (The limits can be increased to meet specific requirements.) Characteristic curves may be multivalued in both current and voltages. Although segments with infinite or nearly infinite slope lead to serious numerical problems, they can be treated accurately if the user is willing to supply an alternate set of equations.



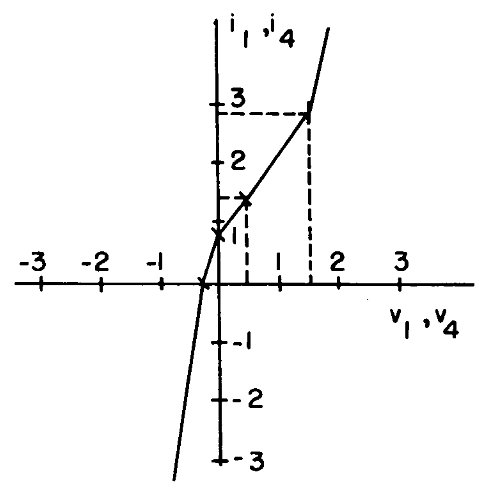
Ia. Nonlinear Lattice



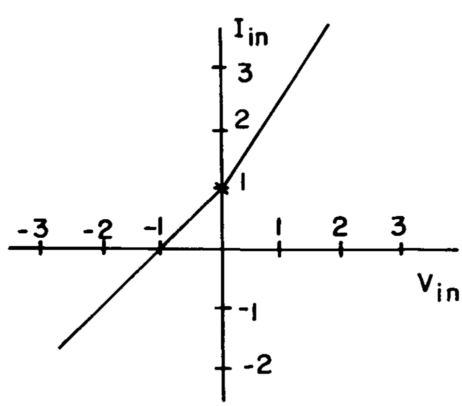
Ib. R5 Characteristic



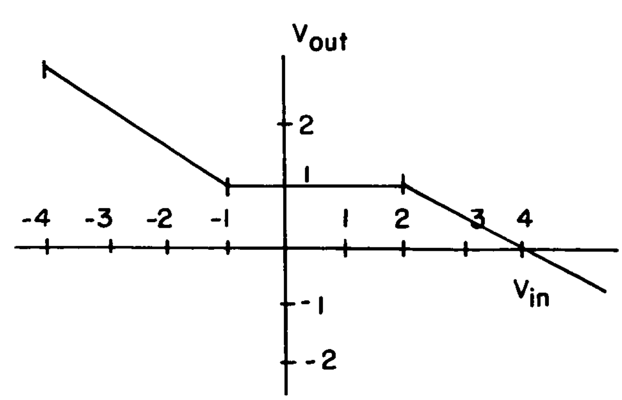
Ic. Series Resistor Characteristics



Id. Shunt Resistor Characteristics



Ie Driving Point Characteristic



If. Transfer Characteristic

Figure I. Data for Example (All Voltages Are in Volts and All Currents in Amperes)

TRANSFER CHARACTERISTIC

OUTPUT VOLTAGE -- INPUT VOLTAGE

```
  *********  +R2  ****
       *           * *
        *+       +*  *
  +      R1      R4        +
  V(IN)  -*     *-   +   V(OUT)
           * *       R5
  -        * *       -     -
          *    *
         *      *    *
        *        *   *
       *          *  *
  *********-R3+  ****
```

SOLUTION SEGMENT	SLOPE	INTERCEPT INPUT / OUTPUT	INTERVAL OF DEFINITION MIN IN / MIN OUT	MAX IN / MAX OUT
2 2 2 2 1 0 0 0 0 0	-0.6667	0.5000 0.3333	-2.5000 1.0000	-1.0000 2.0000
3 3 3 3 1 0 0 0 0 0	-0.	INDETERMINANT 1.0000	-1.0000 1.0000	0. 1.0000
4 4 4 4 1 0 0 0 0 0	-0.	INDETERMINANT 1.0000	0.0000 1.0000	2.0000 1.0000
5 5 5 5 1 0 0 0 0 0	-0.5000	4.0000 2.0000	2.0000 -615382.0547	1230767.2969 1.0000
1 1 1 1 2 0 0 0 0 0	-0.6667	0.5000 0.3333	-750003.3672 2.0000	500002.7422

1245 SEGMENTS UNDEFINED

Figure 2. Computer Output for Transfer Plot. (Figure 1f.)

For a particular problem, the program considers each possible combination of operating states, and selects as valid solutions those for which all equations are consistent. Consider as an example the nonlinear lattice structure in Figure 1a. The five nonlinear resistors are defined graphically in Figures 1b, 1c, and 1d.* While this network is symmetric, symmetry is not a restriction on either the analysis program or the lattice subroutine. Figures 1e and 1f show the driving point plot and input voltage to output voltage transfer characteristic for the circuit, and Figure 2 contains computer output describing the transfer curve. Each piecewise linear segment of the solution is specified in two lines of output data with information arranged as shown in the column headings. The first valid segment is number 2,2,2,2,1, where these digits identify the operating states of resistors R1, R2, R3, R4, and R5 repectively. The slope of the segment is -.6667, and its input and output voltage intercepts are 0.5 and 0.3333 v. respectively. It is defined for $-2.5 \leq V_{in} \leq -1.$v. and 1. v. $\leq V_{out} \leq 2.$ v. Very large positive or negative numbers in the definition intervals indicate that the associated segments extend theoretically to infinity.

If all nonlinear elements in a circuit have strictly monotonically increasing characteristic curves, it is possible, given one segment of the solution, to determine the others without testing every combination of resistor states. The second analysis program was designed for this situation.

---

* This example was constructed with the aid of the lattice synthesis program from information contained in Figures 1b, e, and f. Circuit performance was verified with the general analysis program and was found to be as expected.

Both programs contain a debugging option which prints on request certain constants associated with each valid solution segment to aid the user in detecting errors in the equation subroutine.

A subroutine for the analysis of nonlinear lattice structures has already been written, and another is being developed for ladder networks. In addition the programs are being adapted for use with the CALCOMP model 563 incremental plotter which will provide output directly in graphical form.

REFERENCES

1. Chua, L. O., "Iterative Analysis and Synthesis of Multivalued Memoryless Nonlinear Networks," Proceedings of the National Electronics Conference, Volume XXII, 1966.

2. Chua, L. O., "Iterative Piecewise Linear Analysis of Nonlinear Memoryless Networks," School of Electrical Engineering, Third Semi-Annual Research Summary; "Iterative Piecewise Linear Synthesis of Nonlinear Transfer Characteristics," School of Electrical Engineering, Fourth Semi-Annual Research Summary.

C. FREQUENCY-DOMAIN STABILITY CONDITIONS FOR LINEAR NETWORKS CONTAINING TIME-VARYING RESISTORS, INDUCTORS, AND CAPACITORS

Y. L. Kuo

With the advent of parametric devices and the control problems encountered in space science, there has been considerable recent interest in the theory of systems containing nonlinear time-varying elements. In particular, the need has arisen for stability criteria which can be applied easily in the design of linear and nonlinear time-varying systems.[1,2,3,4] These stability conditions have been sought in terms of the frequency characteristics of the linear time-invariant subsystem and the bounds on the nonlinear and/or time-varying elements. The use of real frequency data results in simplicity and easy application of the stability criteria to higher order systems.

Recently, Sandberg[5] obtained some interesting results concerning the stability of linear networks containing multiple time-varying capacitors. His stability criterion involves calculations of the largest eigenvalue (over entire frequency-domain!) of some positive definite hermitian matrix. This hermitian matrix is a function of the driving-point impedance matrix of the linear, time-invariant n-port network, which is terminated by n time-varying capacitors. In the following, the stability conditions for linear networks containing not just one kind of time-varying elements, but all the three kinds of time-varying elements (i.e., resistors, capacitors, and inductors) are investigated. Stability results involving only n calculations of the successive principle minors of some n x n matrix are obtained, where n denotes the number of time-varying elements. Unlike the results of Sandberg, which become impractical when n is large, our results render simplicity in application (even when n is large), and give some geometrical interpretation for n-port networks as well.

The Main Result

a. Stability Criterion for Linear Networks Containing No Independent Sources.

We shall concern ourselves with the network of Fig. 1 in which a linear, but not necessarily lumped, time-invariant n-port network N is terminated by k linear time-varying resistors, ℓ linear time-varying capacitors and A-k-ℓ linear time-varying inductors. It is assumed that the n-port contains no independent

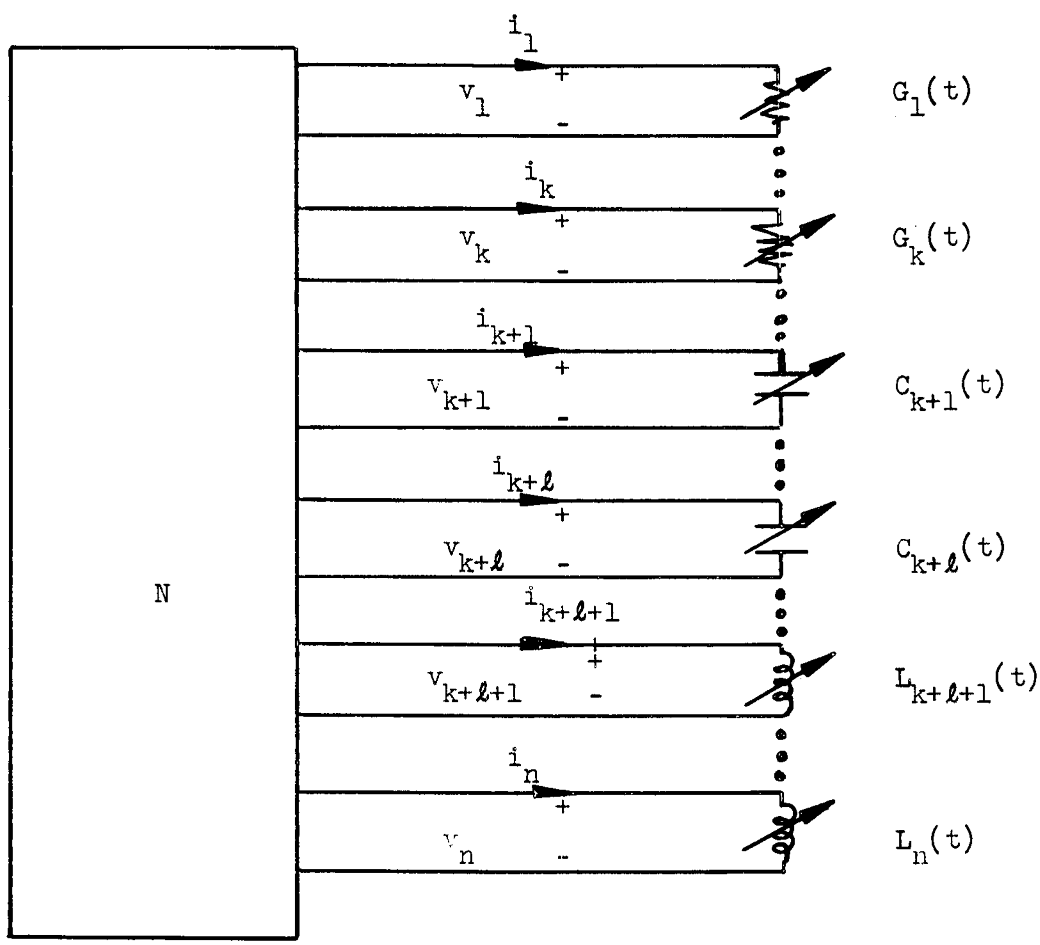


Fig. 1.

An n-port linear time-invariant network N terminated by time-varying resistors, capacitors and inductors.

sources, and all the time-varying conductances $G_1(t)$, ---, $G_k(t)$, capacitances $C_{k+1}(t)$, ---, $C_{k+1}(t)$, inductances $L_{k+\ell+1}(t)$, ---, $L_n(t)$ are sufficiently smooth such that the network possesses a physically meaningful soltuion. We shall make the following assumptions, (A-1) through (A-5):

(A-1) It is assumed that there exist real numbers α_j, β_j $(j=1,---,n)$ such that for $t \geq 0$

$$\alpha_j \leq G_j(t) \leq \beta_j \qquad j=1,---,k$$

$$\alpha_j \leq C_j(t) \leq \beta_j \qquad j=k+1,---,\ k+\ell \tag{1}$$

$$\alpha_j \leq L_j(t) \leq \beta_j \qquad j=k+\ell+1,---,\ n$$

Let

$$G_{oj} = \frac{1}{2}(\beta_j + \alpha_j) \qquad j=1,---,\ k$$

$$C_{oj} = \frac{1}{2}(\beta_j + \alpha_j) \qquad j=k+1,---,\ k+\ell \tag{2}$$

$$L_{oj} = \frac{1}{2}(\beta_j + \alpha_j) \qquad j=k+\ell+1,---,\ n$$

and

$$\hat{G}_j(t) = G_j(t) - G_{oj} \qquad j=1,---,\ k$$

$$\hat{C}_j(t) = C_j(t) - C_{oj} \qquad j=k+1,---,\ k+\ell \tag{3}$$

$$\hat{L}_j(t) = L_j(t) - L_{oj} \qquad j=k+\ell+1,---,\ n$$

Then the network of Fig. 1 is seen to be the same as of Fig. 2, where $\hat{N}$ is the augmented n-port network obtained from N by imbedding the constant conductances G_{o1}, ---, G_{ok} in parallel, the constant capacitances C_{ok+1},---, $C_{ok+\ell}$ in parallel, the constant inductance $L_{ok+\ell+1}$,---, L_{on} in series with the first k ports, the next ℓ ports and the last $(n-k-\ell)$ ports of N respectively.

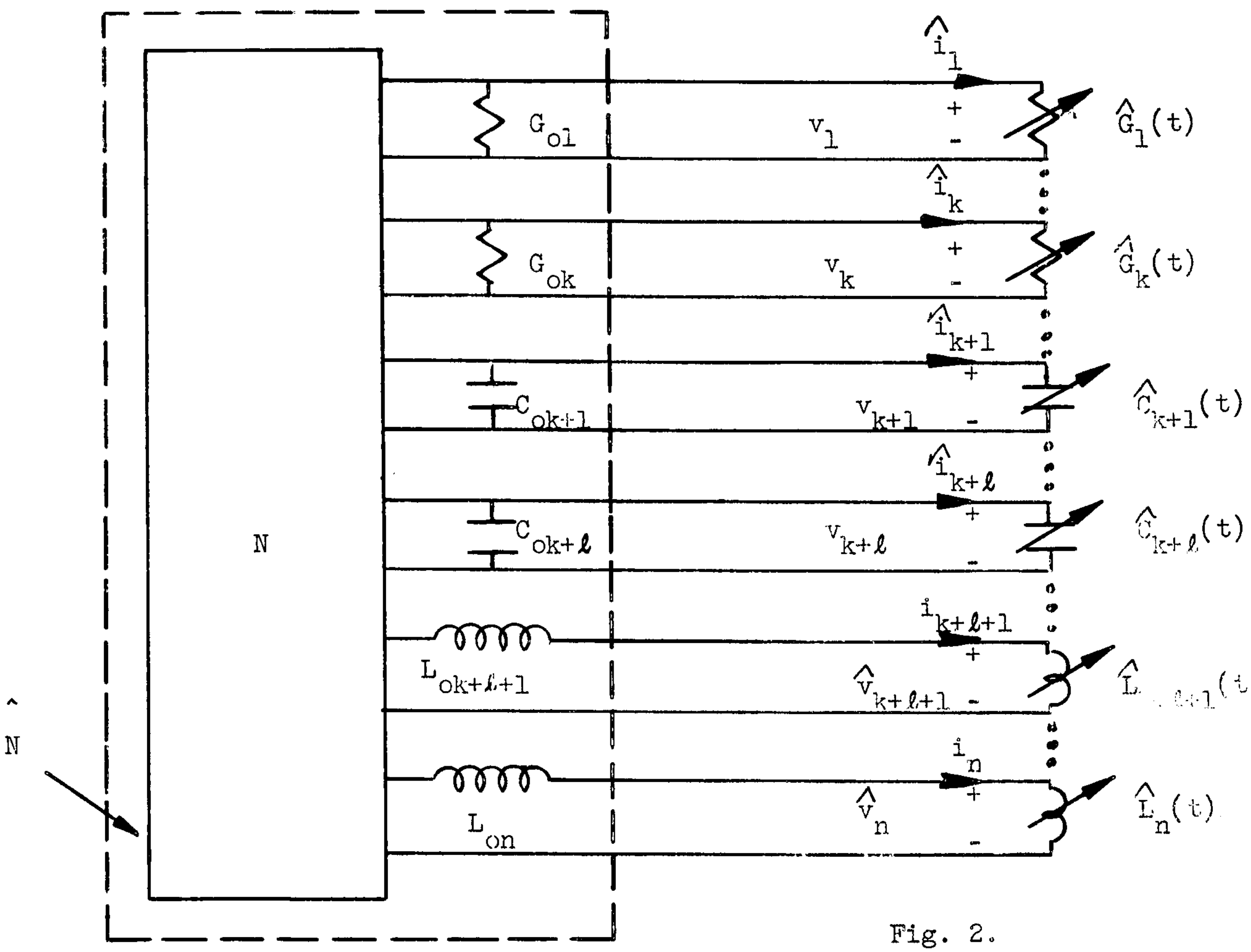


Fig. 2.

In view of (1), (2) , (3) and letting

$$\gamma_j = \frac{1}{2}(\beta_j - \alpha_j), \qquad j=1,---, n \qquad (4)$$

we have

$$|\hat{G}_j(t)| \le \gamma_j \qquad j=1,---, k$$

$$|\hat{C}_j(t)| \le \gamma_j \qquad j=k+1,---,k+\ell \qquad (5)$$

$$|\hat{L}_j(t)| \le \gamma_j \qquad j=k+\ell+1,---, n$$

(A-2) Let

$$\underline{x}(t) = [v_1(t),---,v_k(t), v_{k+1}(t),---, v_{k+\ell}(t), i_{k+\ell+1}(t),---,i_n(t)]'$$

and

$$\underline{y}(t) = [\hat{i}_1(t),---,\hat{i}_k(t), \hat{i}_{k+1}(t),---,\hat{i}_{k+1}(t), \hat{v}_{k+\ell+1}(t),---,\hat{v}_n(t)]'$$

be the output and input n-vectors of $\hat{N}$. Their components $v_j(t)$, $\hat{i}_j(t)$ ($j=1,---,k+\ell$); $\hat{v}_j(t)$, $i_j(t)$($j=k+\ell+1,---,n$) are respectively the port voltages and currents of $\hat{N}$ with polarities as shown in Fig. 2. The main stability criterion to be derived is stated in terms of the properties of the augmented n-port network $\hat{N}$. It is assumed that $\hat{N}$ is characterized by a convolution integral between its input $\underline{y}(t)$ and output $\underline{x}(t)$ of the form:

$$\underline{x}(t) = \underline{x}^o(t) - \int_o^t g(t-\tau)\, \underline{y}(\tau)d\tau \text{ for } + \ge o \qquad (6)$$

where $\underline{x}^o(t) = [v_1^o(t),---, v_k^o(t), v_{k+1}^o(t), v_{k+\ell+1}^o(t),---, i_n^o(t)]'$

is the zero-input response vector of $\hat{N}$, and $g(t) = (g_{ij}(t))(i,j=1,---, n)$ is the unit impulse response matrix of $\hat{N}$.

(A-3) Let $g_{ij}(t) = \tilde{g}_{ij}(t)+d_{ij}\,\delta(t)$ for $i=1,---,n$; $y = 1,---, k$; i.e., elements in the first k columns of the matrix $g(t)$, where d_{ij} are real constants and $\delta(t)$ is the unit impulse function. It is assumed that

(i) $\tilde{g}_{ij}$'s $(i=1,---,n;\ j=1,---,k)$, g_{ij}'s$(i=1,---,n;\ j=k+1,---,n)$ are elements of $L_1(0,\infty) \cap L_2(0,\infty)$

(ii) $\dot{g}_{ij}$'s $(i=1,---,n;\ j=k+1,---,n)$; i.e., the time derivatives of the elements in the last n-k columns of $g(t)$ exist on $[o,\infty)$, and are elements of $L_1(0,\infty) \cap L_2(0,\infty)$. In addition, $g_{ij}(t)$'s $(i=1,---,n;\ j=k+1,---,n)$ tend to zero as $t \to \infty$.

(A-4) It is assumed that for all sets of initial conditions that

(i) v_j^o's $(j=1),---, k+\ell)$, i_j^o's $(j=k+\ell+1,---, 1)$ are elements of $L_2(0,\infty)$

(ii) $v_j^o(t) \to o$ for $j=1,---, k+\ell$ and $i_j^o(t) \to o$ for $j=k+\ell+1,---, n$ as $t \to \infty$

(A-5) It is assumed that the outputs v_j's $(j=1,---,k+\ell)$, i_j's $(j=k+\ell+1,---, n)$ are elements of $L_2(o,T)$ for all finite $T < \infty$; i.e., square integrable over a finite interval.

Definition 1: The network of Fig. 2 is said to be stable* if the following conditions are satisfied for all sets of initial conditions:

(i) v_j's $(j=1,---,k+\ell)$, i_j's $(j=k+\ell+1,---,n)$ are elements of $L_2(o\ \infty)$

(ii) $v_j(t) \to o$ for $j=1,---,k+\ell$ and $i_j(t) \to o$ for $j=k+\ell+1,---, n$ as $t \to \infty$

* Condition (ii) implies that all the port voltages and currents approach zero as $t \to \infty$.

Let

$$\left(\begin{array}{ccc|ccc|ccc}
\hat z_{11}(s) & \cdots & \hat z_{1k}(s) & \hat z_{1,k+1}(s) & \cdots & \hat z_{1,k+\ell}(s) & \hat h_{1,k+\ell+1}(s) & \cdots & \hat h_{1n}(s) \\
\vdots & & & & & & & & \\
\hat z_{k1}(s) & \cdots & \hat z_{kk}(s) & \hat z_{k,k+1}(s) & \cdots & \hat z_{k,k+\ell}(s) & \hat h_{k,k+\ell+1}(s) & \cdots & \hat h_{1n}(s) \\
\hline
\hat z_{k+1,1}(s) & \cdots & z_{k+1,k}(s) & \hat z_{k+1,k+1}(s) & \cdots & z_{k+1,k+\ell}(s) & \hat h_{k+1,k+\ell+1}(s) & \cdots & \hat h_{k+1,n}(s) \\
\vdots & & & & & & & & \\
\hat z_{k+\ell,1}(s) & \cdots & \hat z_{k+\ell,k}(s) & \hat z_{k+\ell,k+1}(s) & \cdots & \hat z_{k+\ell,k+\ell}(s) & \hat h_{k+\ell,k+\ell+1}(s) & \cdots & \hat h_{k+\ell,n}(s) \\
\hline
\hat h_{k+\ell+1,1}(s) & \cdots & \hat h_{k+\ell+1,k}(s) & \hat h_{k+\ell+1,k+1}(s) & \cdots & \hat h_{k+\ell+1,k+\ell}(s) & \hat y_{k+\ell+1,k+\ell+1}(s) & \cdots & \hat y_{k+\ell+1,n}(s) \\
\vdots & & & & & & & & \\
\hat h_{n1}(s) & \cdots & \hat k_{nk}(s) & \hat h_{n,k+1}(s) & \cdots & \hat h_{n,k+\ell}(s) & \hat y_{n,k+\ell+1}(s) & \cdots & \hat y_{nn}(s)
\end{array}\right)$$

be the Laplace transforms of the corresponding elements of $g(t) = (g_{ij}(t))$.

Denote

$$A = \sup_{-\infty<\omega<\infty} \left[\begin{array}{c|c|c} |\hat{z}_{11}(i\omega)|\gamma_1 \cdots |\hat{z}_{1k}(i\omega)|\gamma_k & |\omega\hat{z}_{1,k+1}(i\omega)|\gamma_{k+1} \cdots |\omega\hat{z}_{1,k+\ell}(i\omega)|\gamma_{k+\ell} & |\omega\hat{h}_{1,k+\ell+1}(i\omega)|\gamma_{k+\ell+1} \cdots |\omega\hat{h}_{1n}(i\omega)|\gamma_n \\ \vdots & \vdots & \vdots \\ |\hat{z}_{k+\ell,1}(i\omega)|\gamma_1 \ldots |\hat{z}_{k+\ell,k}(i\omega)|\gamma_k & |\omega\hat{z}_{k+\ell,k+1}(i\omega)|\gamma_{k+1} \cdots |\omega\hat{z}_{k+1,k+\ell}(i\omega)|\gamma_{k+\ell} & |\omega\hat{h}_{k+\ell,k+\ell+1}(i\omega)|\gamma_{k+\ell+1} \cdots |\omega\hat{h}_{k+\ell,n}(i\omega)|\gamma_n \\ \hline |\hat{h}_{k+\ell+1,1}(i\omega)|\gamma_1 \cdots |\hat{h}_{k+\ell+1,k}(i\omega)|\gamma_k & |\omega\hat{h}_{k+\ell+1,k+1}(i\omega)|\gamma_{k+1} \cdots |\omega\hat{h}_{k+\ell+1,k+\ell}|\gamma_{k+\ell} & |\omega\hat{y}_{k+\ell+1,k+\ell+1}(i\omega)|\gamma_{k+\ell+1} \cdots |\omega\hat{y}_{k+\ell+1,n}(i\omega)|\gamma_n \\ \vdots & \vdots & \vdots \\ |\hat{h}_{n1}(i\omega)|\gamma_1 \cdots |\hat{h}_{nk}(i\omega)|\gamma_k & |\omega\hat{h}_{n,k+1}(i\omega)|\gamma_{k+1} \cdots |\omega\hat{h}_{n,k+\ell}(i\omega)|\gamma_{k+\ell} & |\omega\hat{y}_{n,k+\ell+1}(i\omega)|\gamma_{k+\ell+1} \cdots |\omega\hat{y}_{nn}(i\omega)|\gamma_n \end{array}\right] \tag{7}$$

where $\sup\limits_{-\infty<\omega<\infty}$ applies to every element of A.

Our main stability result is stated in the following theorem:

<u>Theorem 1:</u> Let Fig. 2 be the linear time-invariant network under consideration, where $\hat{N}$ is the linear time-invariant part of the network which satisfies the assumptions (A-2) - (A-5), and the linear time-varying conductances $G_1(t)$,---, $G_k(t)$, capacitances $C_{k+1}(t)$,---,$C_{k+\ell}(t)$, inductances $L_{k+\ell+1}(t)$, ---, $L_n(t)$ satisfy the inequality condition (5). If the successive principal minors of (I_n-A) are positive; i.e.,

$$1 - \sup_{-\infty<\omega<\infty} |\hat{z}_{11}(i\omega)|\gamma_1 > 0$$

$$\begin{vmatrix} 1 - \sup |\hat{z}_{11}(i\omega)|\gamma_1 & - \sup\limits_{-\infty<\omega<\infty} |\hat{z}_{12}(i\omega)|\gamma_2 \\ - \sup\limits_{-\infty<\omega<\infty} |\hat{z}_{21}(i\omega)|\gamma_1 & 1 - \sup\limits_{-\infty<\omega<\infty} |z_{22}(i\omega)|\gamma_2 \end{vmatrix} > 0$$

$\cdot \cdot \cdot,$

$$\begin{vmatrix} 1 - \sup\limits_{-\infty<\omega<\infty} |\hat{z}_{11}(i\omega)|\gamma_1 & \cdots & - \sup\limits_{-\infty<\omega<\infty} |\omega\hat{h}_{n}(i\omega)|\gamma_n \\ \vdots & & \\ - \sup\limits_{-\infty<\omega<\infty} |\hat{h}_{n1}(i\omega)|\gamma_1 & \cdots & 1 - \sup\limits_{-\infty<\omega<\infty} |\omega\hat{y}_{nn}(i\omega)|\gamma_n \end{vmatrix} > 0 \qquad (8)$$

then the network is stable in the sense of Definition 1.

Collorary 1: If the sum of the elements in each row (column) of matrix A is less than 1; i.e.,

$$\sum_{p=1}^{k} \sup_{-\infty < \omega < \infty} |\hat{z}_{jp}(i\omega)| \gamma_p + \sum_{p=k+1}^{k+\ell} \sup_{-\infty < \omega < \infty} |\omega \hat{z}_{jp}(i\omega)| \gamma_p +$$

$$\sum_{p=k+\ell+1}^{n} \sup_{-\infty < \omega < \infty} |\omega \hat{h}_{jp}(i\omega)| \gamma_p$$

$$< 1 \text{ for } j = 1,---, k+\ell$$

and

$$\sum_{p=1}^{k} \sup_{-\infty < \omega < \infty} |\hat{h}_{jp}(i\omega)| \gamma_p + \sum_{p=k+1}^{k+\ell} \sup_{-\infty < \omega < \infty} |\omega \hat{h}_{jp}(i\omega)| \gamma_p +$$

$$\sum_{p=k+\ell+1}^{n} \sup_{-\infty < \omega < \infty} |\omega \hat{y}_{jp}(i\omega)| \gamma_p$$

$$< 1 \quad \text{for} \quad j = k+\ell+1,---,n \qquad (9)$$

then the network is stable in the sense of Definition 1.

b. Stability Criterion in Terms of Other Characterizations of the Networks

In the preceding section, the time-varying conductances, capacitances, and inductances are assumed to be bounded for all $t \geq 0$. If, on the other hand, the time-varying elements are bounded resistances, elastances, and inverse inductances such that (A-1) is violated, then the stability criterion expressed in Theorem 1 does not apply. In the following we shall derive stability conditionsin terms of other characterizations of the network so that they apply to our present situation.

We shall consider the network of Fig. 1 except that $G_1(t),--,G_k(t)$, $C_{k+1}(t),---, C_{k+\ell}(t),L_{k+\ell+1}(t),---,L_n(t)$ are now respectively replaced by the time-varying resistances $R_1(t),---,R_k(t)$, the time-varying elastances $S_{k+1}(t)$, $---,S_{k+\ell}(t)$, and the time-varying inverse inductances $\Gamma_{k+\ell+1}(t),---,\Gamma_n(t)$. Similar to the preceding section, we shall make the following assumptions (B-1) - (B-5).

(B-1) It is assumed that there exist real numbers a_j b_j ($j=1,---,n$) such that for $+ \geq o$

$$a_j \leq R_j(t) \leq b_j \qquad j=1,---,k$$

$$a_j \leq S_j(t) \leq b_j \qquad j=k+1,---, k+\ell \qquad (10)$$

$$a_j \leq \Gamma_j(t) \leq b_j \qquad j=k+\ell+1,---, n$$

Let

$$R_{oj} = \frac{1}{2}(b_j + a_j) \qquad j=1,---,k$$

$$S_{oj} = \frac{1}{2}(b_j + a_j) \qquad j=k+1,---, k+\ell \qquad (11)$$

$$\Gamma_{oj} = \frac{1}{2}(b_j + a_j) \qquad j=k+\ell+1,---, n$$

and

$$\hat{R}_j(t) = R_j(t) - R_{oj} \qquad j=1,---, k$$

$$\hat{S}_j(t) = S_j(t) - S_{oj} \qquad j=k+1,---, k+\ell \qquad (12)$$

$$\hat{\Gamma}_j(t) = \Gamma_j(t) - \Gamma_{oj} \qquad j=k+\ell+1,---,n$$

Then the original network is the same as of Fig. 3, where $\hat{N}$ is the augmented n-port network obtained from N by imbedding the constant resistances $R_{o1},---,R_{ok}$ in series, the constant elastances $S_{ok+1},---,S_{ok+\ell}$ in series, the constant inverse inductances $\Gamma_{ok+\ell+1},---,\Gamma_{on}$ in parallel with the first k ports, the next ℓ ports and the least$(n-k-\ell)$ ports of N respectively. In view of (10), (11), (12) and letting

$$\delta_j = \frac{1}{2}(b_j - a_j) \qquad j=1,---,n \qquad (13)$$

we have

$$|\hat{R}_j(t)| \leq \delta_j \qquad j=1,---,k$$

$$|\hat{S}_j(t)| \leq \delta_j \qquad j=k+1,---,k+\ell \qquad (14)$$

$$|\hat{\Gamma}_j(t)| \leq \delta_j \qquad j=k+\ell+1,---, n$$

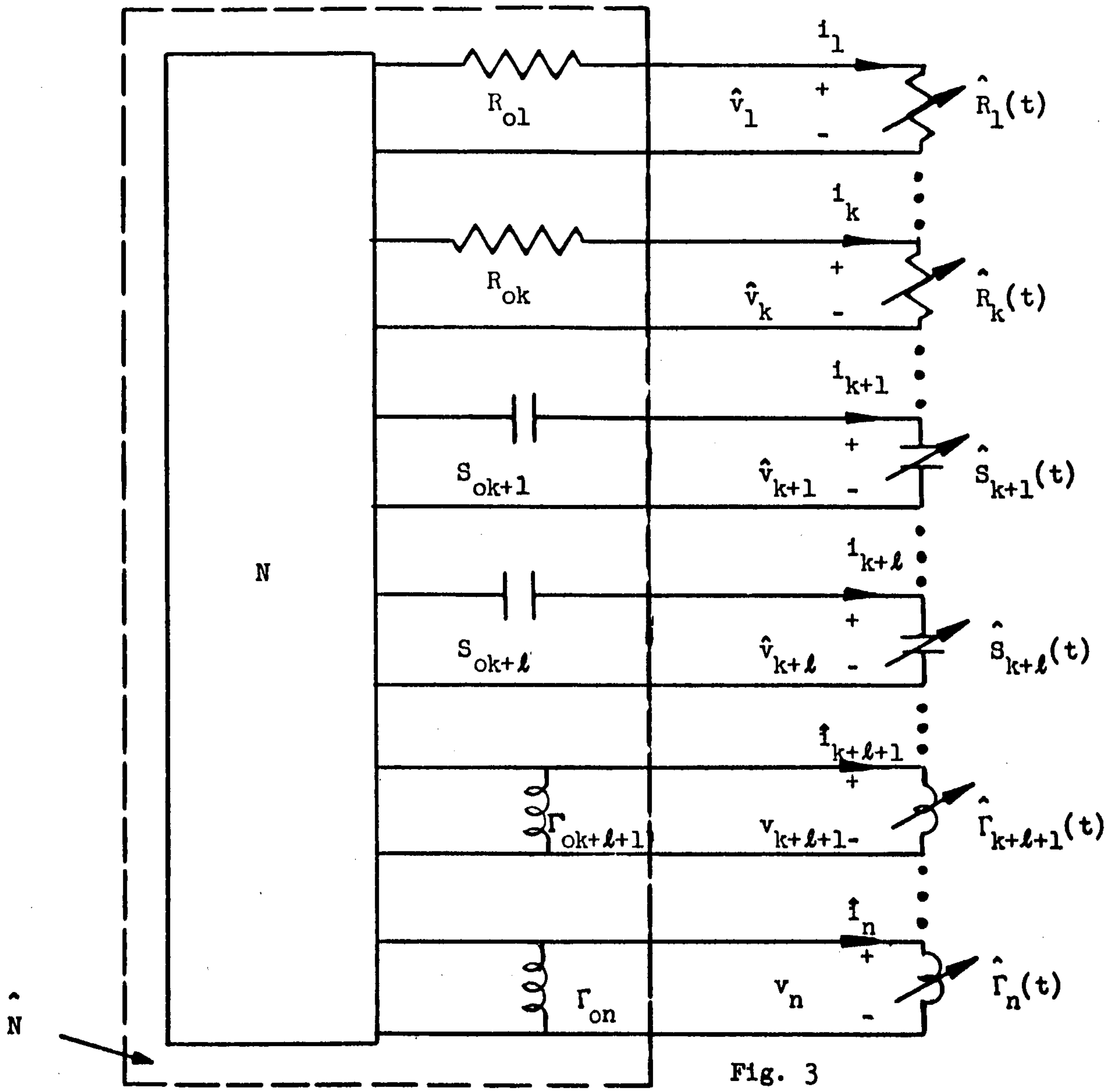


Fig. 3

(B-2) Let

$$\underline{w}(t) = [i_1(t),---,\ i_k(t),\ i_{k+1}(t),\ ---,\ i_{k+\ell}(t), v_{k+\ell+1}(t),---, v_n(t)]'$$

$$\underline{y}(t) = [\hat{v}_1(t),---,\hat{v}_k(t),\ \hat{v}_{k+1}(t),---,\hat{v}_{k+1}(t),\ \hat{i}_{k+\ell+1}(t),---,\ \hat{i}_n(t)]'$$

be the output and input n-vectors of $\hat{N}$. Their components $i_j(t), \hat{v}_j(t)$, $(j=1, ---, k+\ell)$; $\hat{i}_j(t)$, $v_j(t)$, $(j=k+\ell+1,---,n)$ are respectively the port currents and voltages of $\hat{N}$ with polarities as shown in Fig. 3. It is assumed that $\hat{N}$ is characterized by a convolution integral between its input $\underline{y}(t)$ and output $\underline{w}(t)$ of the form:

$$\underline{w}(t) = \underline{w}^o(t) - \int_o^t g(t-\tau)\,\underline{y}(\tau)\,d\tau \text{ for } t \geq o \tag{15}$$

where $\underline{w}^o(t) = [i_1^o(t),---,i_k^o(t),\ i_k^o(t),\ i_{k+1}^o(t),---,i_{k+\ell}(t), v^o_{k+\ell+1}(t),---,$ $v_n^o\ (t)]'$ is the zero-input response vector of $\hat{N}$, and $g(t)=(g_{ij}(t))\ (i,j=1,$ $---,n)$ is the unit impulse response matrix of $\hat{N}$.

(B-3) Let $g_{ij}(t) = \tilde{g}_{ij}(t) + m_{ij}\,\delta(t)$ for $i = 1,---,k;\ j=1,---,n$; i.e., elements in the first kurows of the matrix $g(t)$, where m_{ij} are real constants. It is assumed that

(i) $\tilde{g}_{ij}$'s $(i=1,---,k;\ j=1,---,n)$, g_{ij}'s $(i=k+1,---,n;\ j=1,---,n)$ are elements of $L_1(o,\infty) \cap L_2(o,\infty)$

(ii) $\int_o^t g_{ij}(\tau)\,d\tau$ for $i=k+1,---,n;\ j=1,---,n$ are elements of $L_1(o,\infty) \cap$ $L_2(o,\infty)$

(B-4) Let

$$\underline{x}(t) = [i_1(t),---,i_k(t),\ q_{k+1}(t),---,q_{k+\ell}(t),\ \phi_{k+\ell+1}(t),---,\phi_n(t)]' \tag{16}$$

where $q_{k+1}(t),---,\ q_{k+\ell}(t)$ are the electric changes and $\phi_{k+\ell+1}(t),---,\phi_n(t)$ are the fluxlinkages across the $k+1^{st},---,k+\ell^{th},k+\ell+1^{st},---,$ ports of $\hat{N}$ respectively.

Let

$$\underline{x}^o(t) = [i_1^o(t),---,i_k^o(t),\ q^o_{k+1}(t),---,q^o_{k+\ell}(t),\phi^o_{k+\ell+1}(t),---,\phi_n^o(t)]' \tag{17}$$

be the corresponding zero-input response for $\underline{x}(t)$. It is assumed that for all sets of initial conditions that

(i) i_j^o's $(j=1,\text{---},k)$, q_j^o $(j=k+1,\text{---},k+\ell)$, $\phi_j^o(j=k+\ell+1,\text{---},n)$ are elements of $L_2(o,\infty)$

(ii) $i_j^o(t) \to o$ for $j=1,\text{---},k$, $q_j^o(t) \to o$ for $j=k+1,\text{---},k+\ell$, and $\phi_j^o(t) \to o$ for $j=k+\ell+1,\text{---},n$ as $t \to \infty$.

(B-5) It is assumed that the new outputs $i_1,\text{---},i_k$, $q_{k+1},\text{---}$, $q_{k+\ell},\text{---}$, $\phi_{k+\ell+1},\text{---},\phi_n$ are elements of $L_2(o,T)$ for all finite $T < \infty$.

Definition 2: The network of Fig. 3 is said to be stable* if the following conditions are satisfied for all sets of initial conditions.

(i) $i_1,\text{---},i_k$, $q_{k+1},\text{---},q_{k+\ell}$, $\phi_{k+\ell+1},\text{---}$, ϕ_n are square intergrable.

(ii) $i_1(t),\text{---}$, $i_k(t)$, $q_{k+1}(t),\text{---},q_{k+\ell}(t)$, $\phi_{k+\ell+1}(t),\text{---},\phi_n(t)$ approach zero as $t \to \infty$.

Let

$$\left(\begin{array}{c|c|c}
\hat{y}_{11}(s) \cdots \hat{y}_{1k}(s) & \hat{y}_{1,k+1}(s) \cdots \hat{y}_{1,k+\ell}(s) & \hat{h}_{1,k+\ell+1}(s) \cdots \hat{h}_{1n}(s) \\
\vdots & & \\
\hat{y}_{k1}(s) \cdots \hat{y}_{kk}(s) & \hat{y}_{k,k+1}(s) \cdots \hat{y}_{k,k+\ell}(s) & \hat{h}_{k,k+\ell+1}(s) \cdots \hat{h}_{kn}(s) \\
\hline
\hat{y}_{k+1,1}(s) \cdots \hat{y}_{k+1,k}(s) & \hat{y}_{k+1,k+1}(s) \cdots \hat{y}_{k+1,k+\ell}(s) & \hat{h}_{k+1,k+\ell+1}(s) \cdots \hat{h}_{k+1,n}(s) \\
\vdots & & \\
\hat{y}_{k+\ell,1}(s) \cdots \hat{y}_{k+\ell,k}(s) & \hat{y}_{k+\ell,k+1}(s) \cdots \hat{y}_{k+\ell,k+\ell}(s) & \hat{h}_{k+\ell,k+\ell+1}(s) \cdots \hat{h}_{k+\ell,n}(s) \\
\hline
\hat{h}_{k+\ell+1,1}(s) \cdots \hat{h}_{k+\ell+1,k}(s) & \hat{h}_{k+\ell+1,k+1}(s) \cdots \hat{h}_{k+\ell+1,k+\ell}(s) & \hat{z}_{k+\ell+1,k+\ell+1}(s) \cdots \hat{z}_{k+\ell+1,n}(s) \\
\vdots & & \\
\hat{h}_{n1}(s) \cdots \hat{h}_{nk}(s) & \hat{h}_{n,k+1}(s) \cdots \hat{h}_{n,k+\ell}(s) & \hat{z}_{n,k+\ell+1}(s) \cdots \hat{z}_{nn}(s)
\end{array}\right)$$

* Condition (ii) implies that all the port voltages and currents approach zero as $t \to \infty$.

be the Laplace transforms of the corresponding elements of $g(t) = (g_{ij}(t))$, $i,j=1,\cdots,n$. Let

$$B = \sup_{-\infty<\omega<\infty} \left[\begin{array}{ccc|ccc}
|\hat{y}_{11}(i\omega)|\delta_1 & \cdots & |\hat{y}_{1,k+\ell}(i\omega)|\delta_{k+\ell} & |\hat{h}_{1,k+\ell+1}(i\omega)|\delta_{k+\ell+1} & \cdots & |\hat{h}_{1n}(i\omega)|\delta_n \\
\vdots & & & & & \\
|\hat{y}_{k1}(i\omega)|\delta_1 & \cdots & |\hat{y}_{k,k+\ell}(i\omega)|\delta_{k+\ell} & |\hat{h}_{k,k+\ell+1}(i\omega)|\delta_{k+\ell+1} & \cdots & |\hat{h}_{kn}(i\omega)|\delta_n \\
\hline
\frac{|\hat{y}_{k+1,1}(i\omega)|}{\omega}\delta_1 & \cdots & \frac{|\hat{y}_{k+1,k+\ell}(i\omega)|}{\omega}\delta_{k+\ell} & \frac{|\hat{h}_{k+1,k+\ell+1}(i\omega)|}{\omega}\delta_{k+\ell+1} & \cdots & \frac{|\hat{h}_{k+1,n}(i\omega)|}{\omega}\delta_n \\
\vdots & & & & & \\
\frac{|\hat{y}_{k+\ell,1}(i\omega)|}{\omega}\delta_1 & \cdots & \frac{|\hat{y}_{k+\ell,k+\ell}(i\omega)|}{\omega}\delta_{k+\ell} & \frac{|\hat{h}_{k+\ell,k+\ell+1}(i\omega)|}{\omega}\delta_{k+\ell+1} & \cdots & \frac{|\hat{h}_{k+\ell,n}(i\omega)|}{\omega}\delta_n \\
\hline
\frac{|\hat{h}_{k+\ell+1,1}(i\omega)|}{\omega}\delta_1 & \cdots & \frac{|\hat{h}_{k+\ell+1,k+\ell}(i\omega)|}{\omega}\delta_{k+\ell} & \frac{|\hat{z}_{k+\ell+1,k+\ell+1}(i\omega)|}{\omega}\delta_{k+\ell+1} & \cdots & \frac{|\hat{z}_{k+\ell+1,n}(i\omega)|}{\omega}\delta_n \\
\vdots & & & & & \\
\frac{|\hat{h}_{n1}(i\omega)|}{\omega}\delta_1 & \cdots & \frac{|\hat{h}_{n,k+\ell}(i\omega)|}{\omega}\delta_{k+\ell} & \frac{|\hat{z}_{n,k+\ell+1}(i\omega)|}{\omega}\delta_{k+\ell+1} & \cdots & \frac{|\hat{z}_{nn}(i\omega)|}{\omega}\delta_n
\end{array}\right] \tag{18}$$

Theorem 2: Let Fig. 3 be the linear time-varying network under consideration, where $\hat{N}$ is the linear time-invariant part of the network which satisfies the assumptions (B-2) - (B-5), and the linear time-varying resistances $\hat{R}_1(t), ---, \hat{R}_k(t)$, elastances $\hat{S}_{k+1}(t), ---, \hat{S}_{k+\ell}(t)$, inverse inductances $\hat{\Gamma}_{k+\ell+1}(t), ---, \hat{\Gamma}_n(t)$ satisfy the inequality condition (14). If the successive principal minors of $(I_n - B)$ are positive; i.e.,

$$1 - \sup_{-\infty<\omega<\infty} |\hat{y}_{11}(i\omega)|\,\delta_1 > 0,$$

$$\begin{vmatrix} 1 - \sup\limits_{-\infty<\omega<\infty} |\hat{y}_{11}(i\omega)|\delta_1 & -\sup\limits_{-\infty<\omega<\infty} |\hat{y}_{12}(i\omega)|\delta_2 \\ -\sup\limits_{-\infty<\omega<\infty} |\hat{y}_{21}(i\omega)|\delta_1 & -\sup\limits_{-\infty<\omega<\infty} |\hat{y}_{22}(i\omega)|\delta_2 \end{vmatrix} > 0$$

$$\ldots,$$

$$\begin{vmatrix} 1 - \sup\limits_{-\infty<\omega<\infty} |\hat{y}_{11}(i\omega)|\delta_1 & \cdots & -\sup\limits_{-\infty<\omega<\infty} |\hat{h}_{1n}(i\omega)|\delta_n \\ \vdots & & \\ -\sup\limits_{-\infty<\omega<\infty} \left|\dfrac{\hat{h}_{n1}(i\omega)}{\omega}\right|\delta_1 & \cdots & 1 - \sup\limits_{-\infty<\omega<\infty} \left|\dfrac{\hat{z}_{nn}(i\omega)}{\omega}\right|\delta_n \end{vmatrix} > 0 \qquad (19)$$

then the network is stable in the sense of Definition 2.

Collarary 2: If the sum of the elements in each row (column) of matrix B is less than 1; i.e.,

$$\sum_{p=1}^{k+\ell} \sup_{-\infty<\omega<\infty} |\hat{y}_{jp}(i\omega)|\delta_p + \sum_{p=k+\ell+1}^{n} \sup_{-\infty<\omega<\infty} |\hat{h}_{jp}(i\omega)|\delta_p < 1 \text{ for } j=1,---,k$$

$$\sum_{p=1}^{k+\ell} \sup_{\omega} \left|\frac{\hat{y}_{jp}(i\omega)}{\omega}\right|\delta_p + \sum_{p=k+\ell+1}^{n} \sup_{\omega} \left|\frac{\hat{h}_{jp}(i\omega)}{\omega}\right|\delta_p < 1 \text{ for } j=k+1,---,k+\ell$$

and

$$\sum_{p=1}^{k+\ell} \sup_{\omega} \left|\frac{\hat{h}_{jp}(i\omega)}{\omega}\right|\delta_p + \sum_{p=k+\ell+1}^{n} \sup_{\omega} \left|\frac{\hat{z}_{jp}(i\omega)}{\omega}\right|\delta_p < 1 \text{ for } j=k+\ell+1,---,n \tag{20}$$

then the network is stable in the sense of Definition 2.

For a linear, time-invariant n-port network, one may have up to 2^n different ways of characterizing the network in terms of impedances, admittances, and hybrid parameters. To each characterization there corresponds a stability criterion similar to the ones in Theorems 1 and 2. Except for some special cases, the stability criteria obtained from different characterizations may not give the same results. One form of stability criterion may be better than others for some networks, while it may be worse than others for other networks. However, for one-port case, it can be shown that Theorems 1 and 2 give the same stability result.

The details of the proofs will appear elsewhere.

c. Some Geometrical Interpretation of Theorems 1 and 2

Let us first consider the time-varying network of Fig. 2. Theorem 1 states that the network is stable if all the successive principal minors of (I_n-A) are positive. Thus, in order to satisfy the stability criterion, first of all the diagonal elements of (I_n-A) must be positive. The geometrical interpretation of this condition is rather interesting. Indeed, consider the first diagonal element,

$$1 - \sup_{-\infty < \omega < \infty} |\hat{z}_{11}(i\omega)| \gamma_1 > 0 \tag{21}$$

This guarantees the stability of the network if all the time-varying elements except $\hat{G}_1(t)$ are removed from $\hat{N}$ (i.e., opening $\hat{G}_2(t)$,---,$\hat{G}_k(t)$, $\hat{C}_{k+1}(t)$, ---, $\hat{C}_{k+\ell}(t)$ and short-circuiting $\hat{L}_{k+\ell+1}(t)$,---, $\hat{L}_n(t)$). Eq (21) says that the 1-port network so obtained is stable if the locus of $\hat{z}_{11}(i\omega)$ lies inside a circle with radius $1/\gamma_1$ and centered at the origin of the complex plane. This can be shown to be equivalent to the well-known circle condition of Sandberg[4]. We conclude that in order to satisfy the stability criterion for the n-port network, it is necessary that every one-port obtained by 'removing' the rest of the time-varying elements must be stable. This is intuitively reasonable, since there is no sense in considering the stability of the n-port network if one of the 1-port networks so obtained does not satisfy its own stability criterion.

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D. FREQUENCY-DOMAIN CRITERIA FOR THE STABILITY OF NONLINEAR SYSTEMS

Y. L. Kuo

L. O. Taylor

Progress has been made on a literature search in the general area of frequency domain criteria for the stability of nonlinear systems. The existing methods are in many cases difficult to apply to problems of a practical nature and very restrictive on the linear part of the system, while allowing a great deal of freedom with regard to the nonlinearity[1, 2, 3]. For multiple nonlinearities the existing results are very difficult to apply, requiring large amounts of computation before obtaining useful information[1].

It is desirable to have a criterion for stability that is easy to apply and permits a trade-off between the restrictions on the linear and nonlinear portions of a system.

The following areas are being investigated with this goal in mind:

1. Bounded input, bounded output stability.

   Most of the existing results determine Liapunov stability. The more difficult, and perhaps more useful from an engineering point of view, problem of determining when the response of a system is bounded for bounded inputs is largely unsolved.

2. L_2 stability

   This problem is rather loosely defined as determining the restrictions under which a system is stable in the sense that the response is square integrable. Some rather general results have been obtained in this area for systems with single or multiple nonlinearities. (1) However, the results for multiple nonlinearities are very difficult to apply. Effort is being made to determine more conveniently applied criteria, and perhaps extend the region of stability.

3. Stability for systems restricted to one piecewise linear nonlinearity.

   This quite practical problem appears to have received little attention in the literature, the more general problems being more popular. Results here could be quite helpful in design procedures.

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E. NEGATIVE RESISTANCE AMPLIFIERS WITH PRESCRIBED DELAY

B. J. Leon

G. T. Cotter

The problem of synthesizing a passive network that meets separate magnitude and phase specifications over a prescribed band has been studied by several investigators.[1-7] With active networks becoming as easy to construct as passive networks, it seems appropriate to re-evaluate the basic problem of independent magnitude and phase specification without the restrictions of passivity. Methods of synthesizing active networks using circulators and negative resistors in addition to positive RLC are being studied by several groups.[8,9,10] This configuration is the first that is being investigated.

By using the potential analogue method, Kuh[2] has designed a low pass ladder which shapes the response while an all-pass bridge, connected in tandem, gives the desired time delay. Hosking[3] has shown that n of these bridges, connected in tandem, will result in n times the delay of the original network.

Weak variations of the time delay in any frequency interval have been extensively examined by Ulbrich and Piloty[4]. Their synthesis approach, applicable to low, band, and high pass filters, yields an approximate delay by using Tschebycheff rational polynomials.

Colin and Allemandou[5], by utilizing complex conjugate pole pairs, have obtained time delay without distortion in the passband. Allemandou[6] extended the method, and successfully synthesized a band pass filter, by realizing that flat delay at a given frequency implies zero time zero time delay first derivatives for this frequency.

Maximally flat time delay has been achieved by Deutch[7] with lowpass ladders, where accuracy is proportional to the number of these ladder networks.

The general matching problem was considered by Chan and Kuh[8] for an active or passive (but not lossless) load. A reflection type amplifier was synthesized by connecting a load, through a lossless coupling network, to a circulator. Results with a tunnel diode as load are given. The tunnel diode problem has also been investigated by Monaco.[9]

Fjallbrant[10] has presented a detailed synthesis procedure using the Smith chart. His method is applicable to any frequency range. He further derives a relationship enabling the synthesis of networks with prescribed input impedance.

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F. TRANSIENT DECAY IN NONLINEAR NETWORKS

D. R. Anderson

T. C. Huang

The theory of the linear time-invariant systems has almost been fully developed, and current researchers have begun to turn their attention to the time-varying and/or the nonlinear problems. Yet, in analyzing the latter systems, there is not any effective mathematical method, though engineers have successfully used the systems. Fortunately, linearization techniques have been devised and they not only have solved quite a few problems, but also have provided some methods for designing a system. However, the linearization can not completely account for all the nonlinear phenomena, such as the jump phenomenon and subharmonics. Nowadays, engineers have been increasingly concerned about these anomalous phenomena. Thus it is necessary to study the behavior of the nonlinear system.

Usually the behavior of a system may be analyzed in terms of the steady state and the transient responses. For linear systems these two responses have been well understood, and the steady state response is generally unique. But the

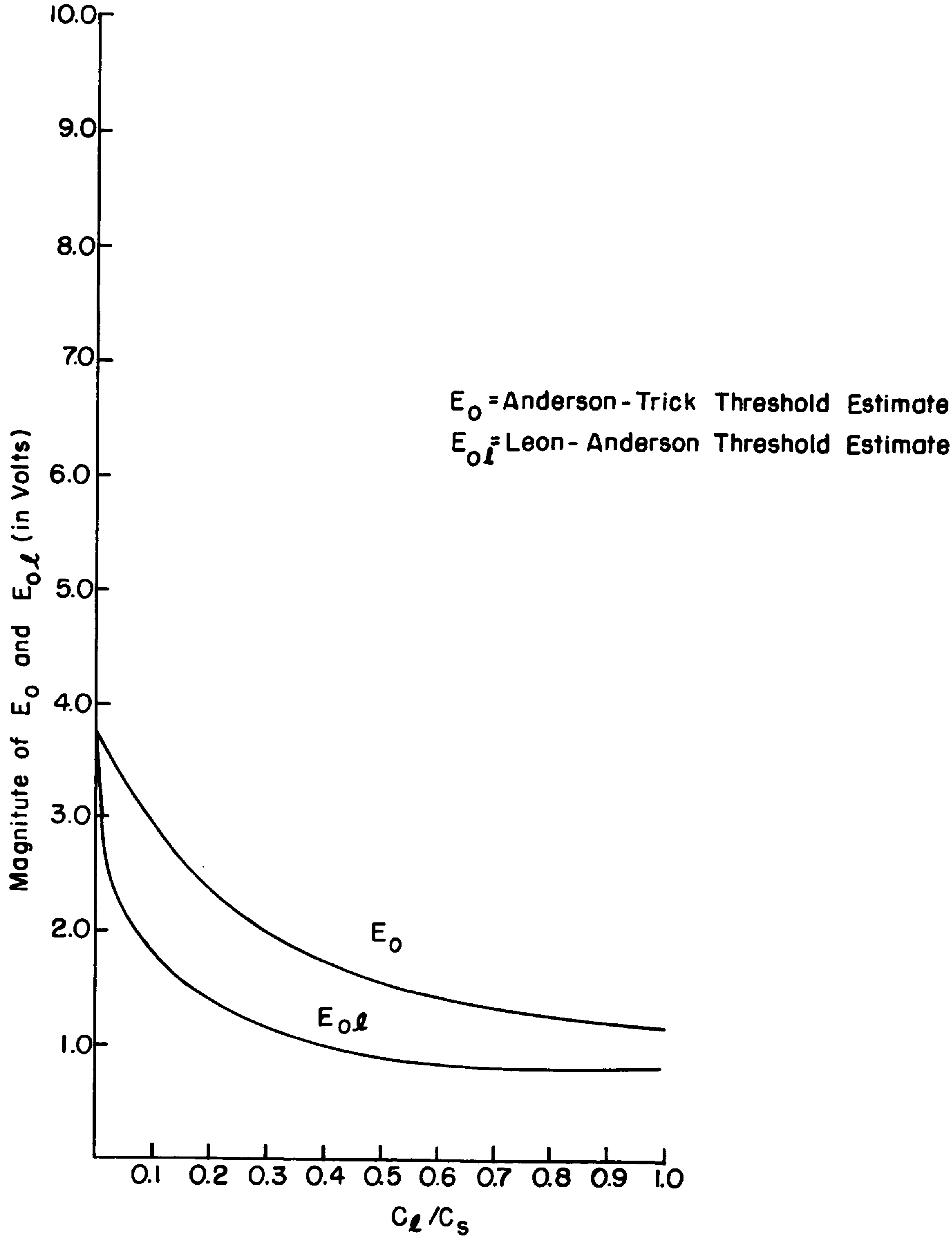

Figure 1 The Comparison of the Two Methods of Estimation

latter uniqueness does not always exist in nonlinear systems. Thus in nonlinear systems, the transient response is of interest not only from the standpoint of the rate of decay, but also from the standpoint of the particular steady state achieved. It is hoped that through our formulations the transient behaviors will be studied successfully.

It is known that a nonlinear system can be characterized by the differential, difference, integral, or integro-differential equations. In this proposal only the integral equation approach will be discussed, because this approach is more convenient for handling.

This study considers two approaches to the study of the transient behavior of the nonlinear system. The first one is based on Leon and Anderson's work[1]. Using Anderson and Trick's result[2] which provides, so far, the best method for estimating the periodic steady state response, the second one is to formulate the transient response by subtracting the steady state response from the system response. To illustrate that the second formulation is better than the first, the estimation on the transient response from both formulations and an example are worked out to get a concrete comparison of the two methods of estimation. As is shown in the graph, Figure 1, the second method does indeed provide much better information about the transient behavior.

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G. GENERATION OF CONSTANT RESISTANCE NETWORKS BY TOPOLOGICAL METHODS

P. M. Lin

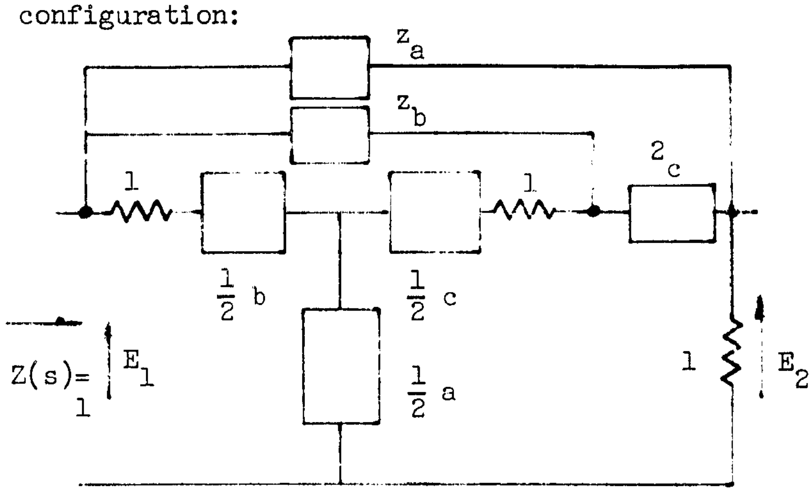
Constant resistance networks play a very important role in information transmission systems. They are used extensively in attenuation equalizers, delay equalizers, and filter pairs. Zadeh[1] was the first one to point out that any self-dual network, whether fixed or linear time-varying, is a constant resistance network. Very recently, Desoer, Newcomb, and Wong generalized the result and give a precise condition for the truth of the statement "every self-dual one-port is constant resistance and conversely every constant resistance one-port is self-dual".

As far as the network configuration is concerned, the only self-dual constant resistance networks known to date are the classical lattice and bridged-T networks. All other constant resistance networks (including the full-series and full-shunt constant resistance networks) can be derived from these basic structures through network transformations and equivalences.

We undertake to investigate the topological aspect of the problem with the aim of obtaining new constant resistance network configurations which are derivable from the existing ones. The following theorem is proved:

"Let $G\rho$ be a self-dual one terminal-pair graph with respect to vertices (i,j), and with the degrees of (i,j) both equal to ρ. For $\rho \geq 2$, $G\rho$ cqn be realized with $8\rho - 11$ edges, but not with fewer edges, if the union of $G\rho$ and an edge joining (i,j) is to be 3-connected."

Based on the graphs of the above theorem, a wide class of constant resistance networks is generated. For $\rho = 2$, the graphs lead to the classical lattice and bridged-T constant resistance networks. The network obtained with $\rho = 3$, after some further manipulation, leads to the following interesting configuration:



$$\frac{E_2}{E_1} = \frac{(1+z_b)(1+z_c)+z_a-1}{(1+z_a)(1+z_b)(1+z_c)-1}$$

This configuration has been found useful in transfer function synthesis. Full report of the topological approach mentioned here will appear elsewhere. Currently, research is continued to find out other useful properties of the higher order ($\rho \geq 3$) constant resistance networks.

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II. NEW NETWORK ELEMENTS

N67-33612

A. THE SCALOR -- A NEW NETWORK ELEMENT

L. O. Chua

1. Introduction

The systematic synthesis of nonlinear network elements requires the following three basic transformations:

1) translation
2) rotation
3) scaling

The first transformation can be easily implemented by independent sources. The second transformation can be implemented by a Rotator. In order to implement the third transformation, we need a new network element -- the Scalor.

2. Definitions

We define three classes of Scalors according to the following transmission (Chain) matrices:

Voltage Scalor (V-Scalor)

$$\underset{\sim}{T}(V) = \begin{bmatrix} K_V & 0 \\ 0 & 1 \end{bmatrix}$$

where K_V is a constant, hence forth called the voltage scaling constant.

Current Scalor (I-Scalor)

$$\underset{\sim}{T}(I) = \begin{bmatrix} 1 & 0 \\ 0 & K_I \end{bmatrix}$$

where K_I is a constant, henceforth called the current scaling constant.

Power Scalor (P-Scalor)

$$\underset{\sim}{T}(P) = \begin{bmatrix} K_P & 0 \\ 0 & K_P \end{bmatrix}$$

where K_P is a constant, henceforth called the power scaling constant.

3. Properties of Scalors

1. Tl a nonlinear resistor, inductor, or capacitor is connected across port 2 of a V-scalor, we obtain a new resistor, inductor, or capacitor characterized by the original I-V, ∅-I or 2-V curve but with a different scale factor. In particular, the voltage of the resistor, the flux-linkage of the inductor, or the voltage of the capacitor is multiplied by K_V respectively.
2. For an I-scalor, property 1 applies to the current of the resistor, the current of the inductor, or the charge of the capacitor.
3. For a P-scalor, property 1 applies to both current and voltage of the resistor, the flux-linkage and current of the inductor, or the charge of the capacitor.
4. If we let $K_V=K_I=K_P$, and connect a V-scalor in cascade with an I-scalor, or vice versa, we obtain a P-scalor. This is easily seen since $\underset{\sim}{T}(V)\ \underset{\sim}{T}(I) = \underset{\sim}{T}(I)\ \underset{\sim}{T}(V) = \underset{\sim}{T}(P)$.

4. Realization of the Scalor

Formally, a V-scalor can be realized by the controlled source model shown in Figure 1, and an I-scalor can be realized by the controlled source model shown in Figure 2. In view of property 4, a P-scalor can be formally realized by connecting the above networks in cascade.

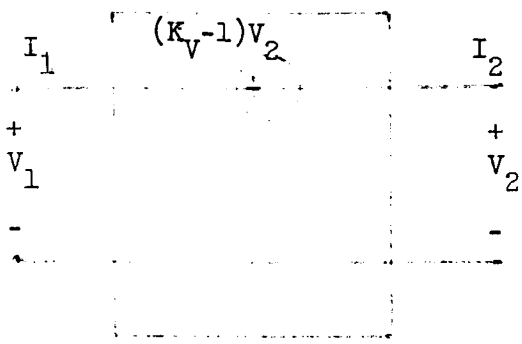


Fig. 1

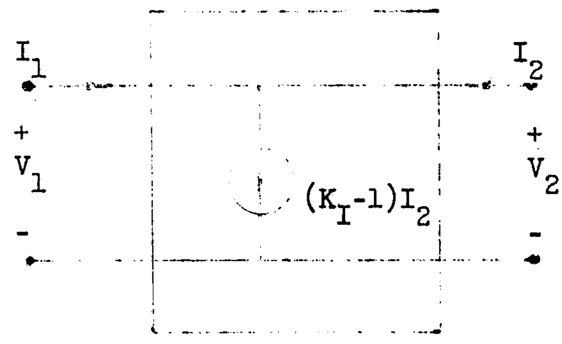


Fig. 2

Scalors can be realized in practice by various solid state circuits. Some of these will be given in the next Semi-Annual report.

B. A NEW LOOK AT THE ROTATOR

L. O. Chua

1. A New Characterization

In order to avoid ambiguity, a Rotator must be described by two parameters: an angle of rotation θ, and a scale factor K. To distinguish the three different types of Rotators, we will denote the two parameters by (θ,R) for an R-Rotator, or (θ,L) for an L-Rotator, or (θ,C) for a C-Rotator. The new characterization assumes the form:

R-Rotator

$$V_1 = (\cos\theta)\, V_2 + R\,(\sin\theta)\, I_2$$
$$I_1 = (1/R)(\sin\theta)\, V_2 - (\cos\theta)\, I_2$$

L-Rotator

$$\phi_1 = (\cos\theta)\,\phi_2 + L\,(\sin\theta)\,I_2$$

$$I_1 = (1/L)\,\sin\phi_2 - (\cos\theta)\,I_2$$

C-Rotator

$$Q_1 = -\,(\cos\theta)\,Q_2 - c(\sin\theta)\,V_2$$

$$V_1 = -\,(1/c)(\sin\theta)\,Q_2 - (\cos\theta)\,V_2$$

The corresponding sets of 2-port parameters are listed in Table 1.

2. New Applications of the Rotator

The Rotator was originally conceived as a basic component for synthesizing nonlinear elements characterized by multivalued curves. In order to illustrate the necessity for using Rotators as basic building blocks, we will describe briefly several practical synthesis procedures for realizing multivalued I-V curves[1]. The same concepts are obviously also applicable to the synthesis of multivalued ϕ-I and Q-V curves.

(a) Synthesis of Strictly Monotonically Increasing I-V Curves

Several well-known procedures exist for realizing strictly monotonically increasing I-V curves[2,3]. The basic principle involves the successive series and parallel combinations of two basic building blocks whose I-V curves are shown respectively in Fig. 1(a) and (b). We will refer to any 2-terminal network having the I-V curve shown in Fig. 1(a) or Fig. 1(b) as a "concave" or "convex" resistor respectively. Until now, concave and convex resistors are realized by using junction diodes, linear resistors, and batteries[2,3]. The need for using batteries greatly reduces the practical value of the above procedure. We will now show how the batteries can be eliminated. By making use of the I-V curve of a typical "zener diode" as shown in Fig. 1(c) and the I-V curve of a typical "field effect diode" as shown in Fig. 1(d), we can realize any concave resistor by the circuit shown in Fig. 1(e), and any convex resistor by the circuit shown in Fig. 1(f).

(b) Synthesis of Current Controlled and Voltage Controlled I-V Curves

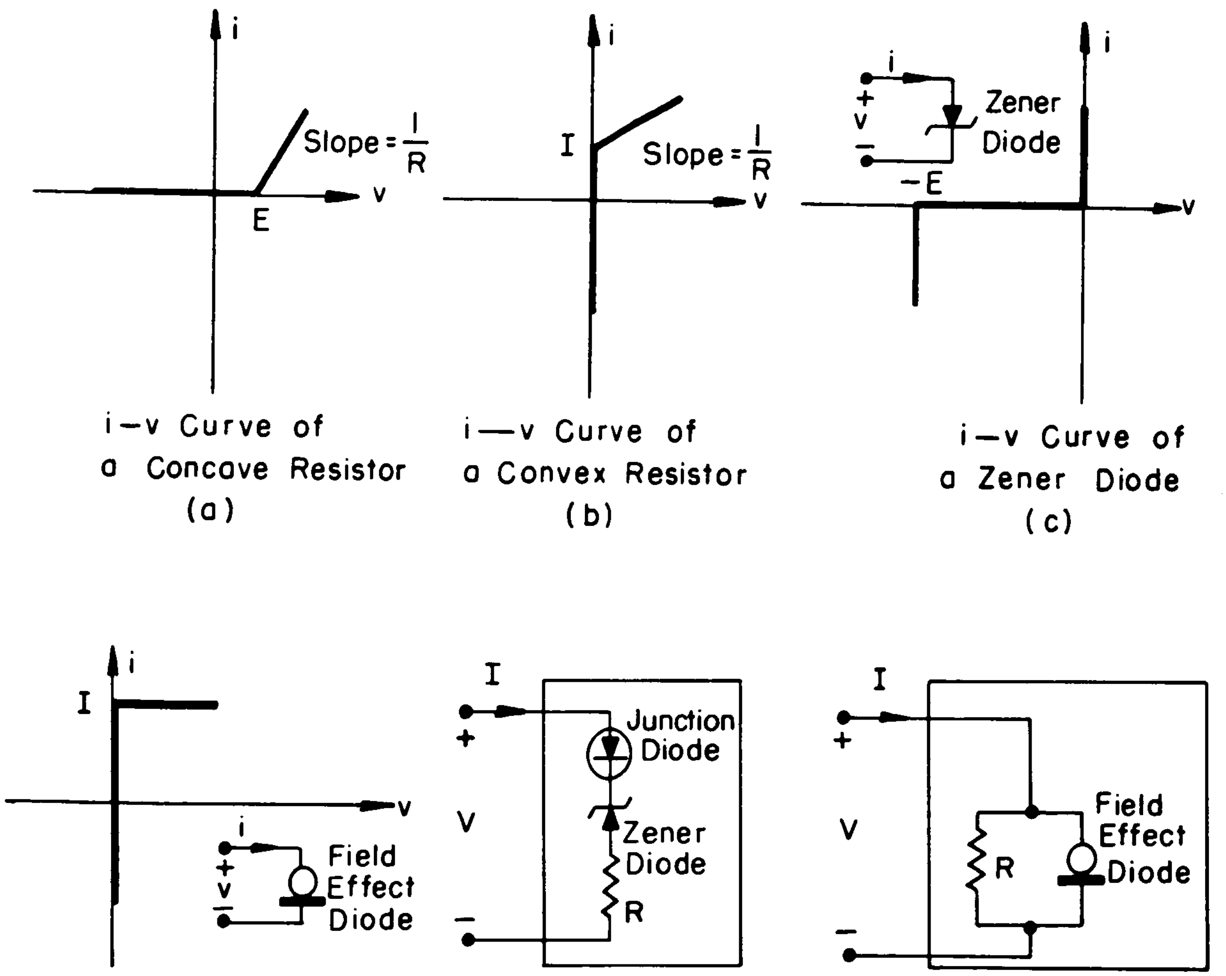
It is well know that any current (voltage) controlled I-V curve can be realized by connecting a suitable negative resistance in series (parallel) with a nonlinear resistor characterized by a strictly monotonically increasing I-V curve. This shows that any current (voltage) controlled I-V curve can be realized by using only concave resistors, convex resistors, and one negative resistance.

(c) Synthesis of Multivalued I-V Curves

Until now, no general procedure is known for realizing multivalued I-V curves without using controlled sources[1]. We will show that with the help of R-Rotators, virtually any multivalued I-V curve which does not intersect itself can be realized without using any controlled source. We will consider simpler, multivalued curves first.

For convenience, a multivalued I-V curve will be called a "quasi-multivalued curve" if we can transform it into a current controlled or voltage controlled I-V curve by a rotation about the origin. A quasi-multivalued curve can be easily realized by synthesizing first a current controlled or a voltage controlled I-V curve, and then rotating the resulting curve by an R-Rotator as shown in Fig. 2. For example, the I-V curve shown in Fig. 3 is a quasi-multivalued curve, and therefore can be realized by the above method.

A more general class of multivalued I-V curves is the class of "Strictly Unicursal Curves" as defined in reference 1. Any strictly unicursal I-V curve can be synthesized by using only concave resistors, convex resistors, and R-Rotators. This result is based on the observation that if we connect a concave resistor across one port of an R-Rotator, we obtain a set of secondary building blocks with I-V curves as shown in Figs. 4(a) and (b). Similarly, if we connect a convex resistor across one port of an R-Rotator, we obtain a set of secondary building blocks with I-V curves as shown in Figs. 5(a) and (b). The above sets of secondary building blocks are more than enough to synthesize any strictly unicursal curve[1], upon replacing the terms "current controlled curves" and "voltage controlled curves" in the definition by the less restrictive term "quasi-multivalued curves." Clearly, any multivalued curve which can be reduced to a strictly unicursal curve by a rotation about the origin belongs to the above class. For example, consider the multivalued curve shown in Fig. 8(a). Although this curve is not strictly unicursal, it can be rotated by θ° about the origin to obtain the strictly unicursal curve shown in Fig. 8(b). Hence the curve shown in Fig. 8(a) can be realized by the network shown Fig. 8(c).



i−v Curve of a Concave Resistor (a)

i—v Curve of a Convex Resistor (b)

i−v Curve of a Zener Diode (c)

i- v Curve of a Field Effect Diode (d)

A Practical Realization of a Concave Resistor (e)

A Practical Realization of a Convex Resistor (f)

Figure 1

Basic Components For Realizing Monotonically Increasing I−V Curves

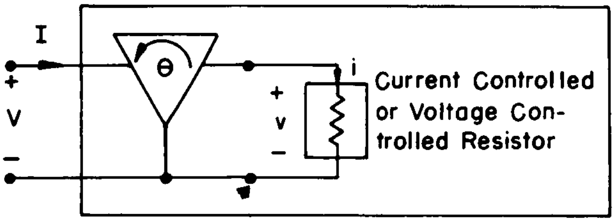


Figure 2

Synthesis of Quasi-Multivalued I−V Curves

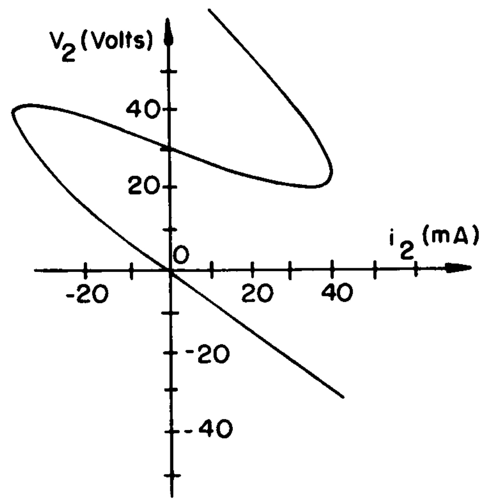


Fig. 3 I-V Curve

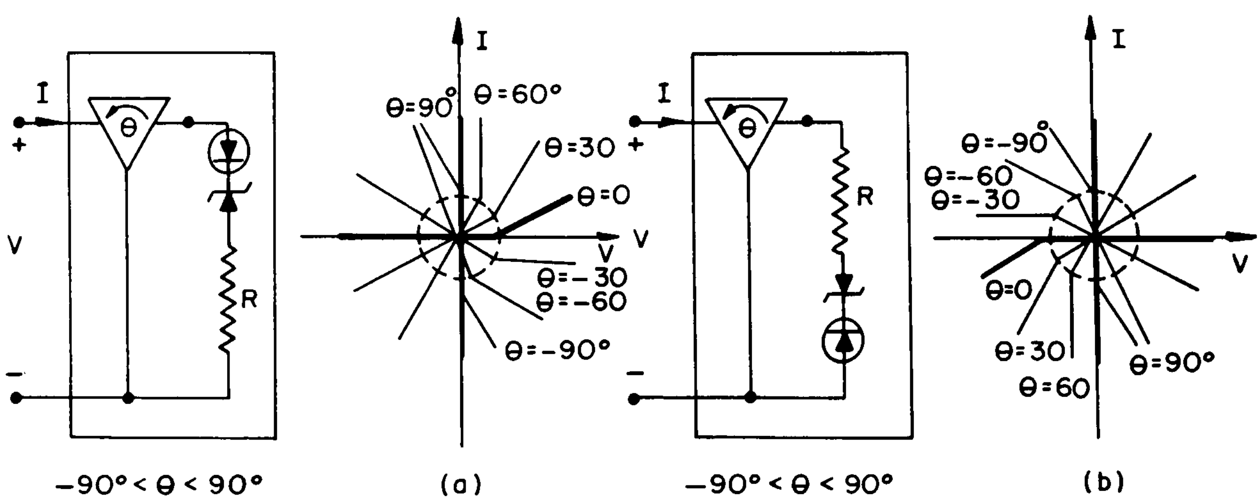


Figure 4

Practical Realization of a Set of Secondary Building Blocks by Using a Rotator and a Concave Resistor

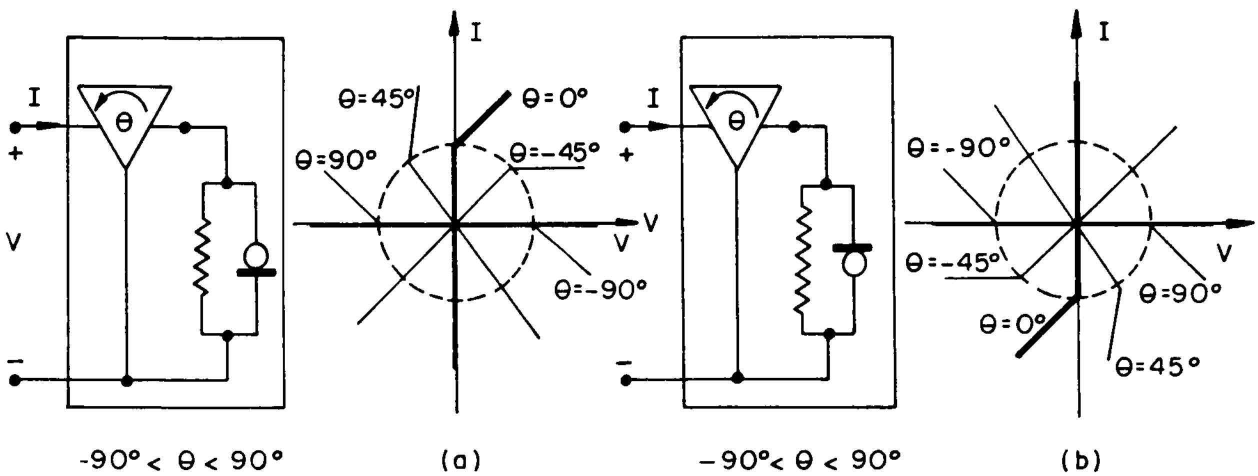


Figure 5

Practical Realization of a Set of Secondary Building Blocks by Using a Rotator and a Convex Resistor

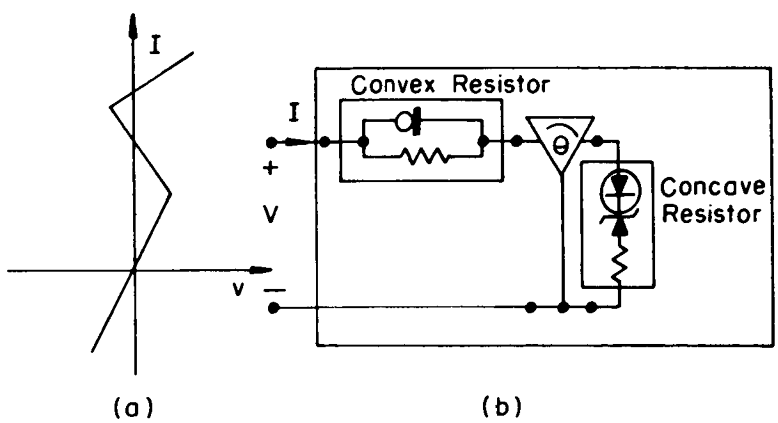


Figure 6

Realization of a Current Controlled I-V Curve

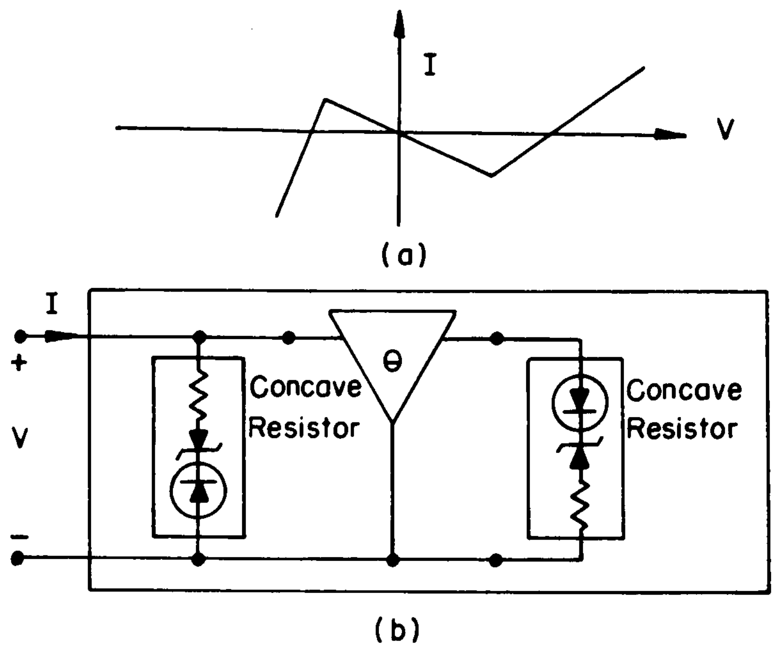


Figure 7
Realization of a Voltage Controlled I-V Curve

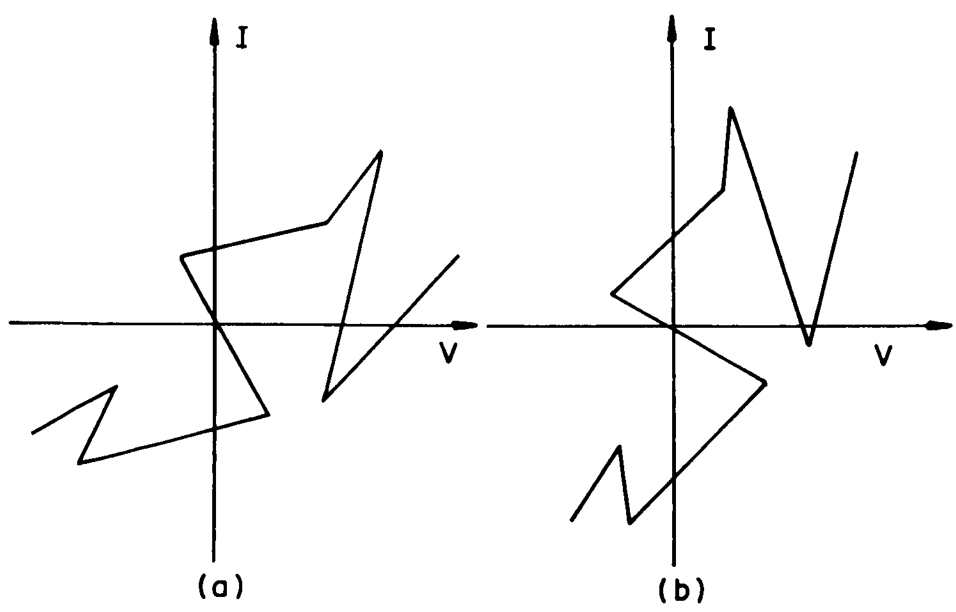


Figure 8
A Multivalued Curve Which Can Be Rotated to Obtain a Strictly Unicursal Curve

By a combination of the above techniques, it is readily seen that virtually any univursal curve which does not intersect itself can be realized by a successive series and parallel combinations of concave resistors, convex resistors, and R-Rotators.

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C. ROTATOR FREQUENCY RESPONSE

D. A. Perreault

L. O. Chua

Before measuring the "frequency response" of a Rotator[1,2], it is necessary to discuss the meaning of the term frequency response. When in operation, the Rotator is a nonlinear one-port device; therefore the normal definition of frequency response is not applicable.

Since the purpose of a Rotator is to rotate a device's characteristic curve through some prescribed angle, it would be meaningful to define the response as the relation between the angle of rotation and the driving frequency. This would present a family of curves, each one being specified by the desired angle of rotation. To avoid any confusion between this response and the normal definition of frequency response, the Rotator's response will be referred to as the Rotator Frequency Characteristic.

To measure the Rotator Frequency Characteristic, it is necessary to select some standard to be rotated; for convenience, a linear element was chosen. In the case of the R-Rotator, a 1000 linear resistor is chosen so that any distortion of the test element will be readily noticeable.

The measuring circuit shown in Fig. 1 was used in making all measurements. The compensation network was used only for frequencies above 10 KH_z. This network was used to compensate for the output reactance of the oscillator at high frequencies, and was utilized in the following manner: the linear test resistor was connected across the measuring circuit; then at a given frequency, various R, L, C networks were used to compensate for any phase shift seen on the oscilloscope's I-V trace. This R, L, C combination was then used as a compensating network, and the Rotator was connected back into the circuit to measure its response.

The actual rotation angles were measured in the ma-volt plane as follows: First, the slope of the test resistor was measured, and its angle defined as

$$\psi = \text{Arctan}\left(\frac{\Delta I}{\Delta V}\right).$$

Secondly, the slope after rotation was measured, and the angle after rotation was

$$\phi = \text{Arctan}\left(\frac{\Delta I}{\Delta V}\right).$$

Since the angle of rotation (θ) is the difference between the intial and the final angles, then

$$\theta = \phi - \psi.$$

ψ was measured for various preset angles of rotation and frequencies, and from this data θ was computed and plotted as shown in Figs. 2 and 3.

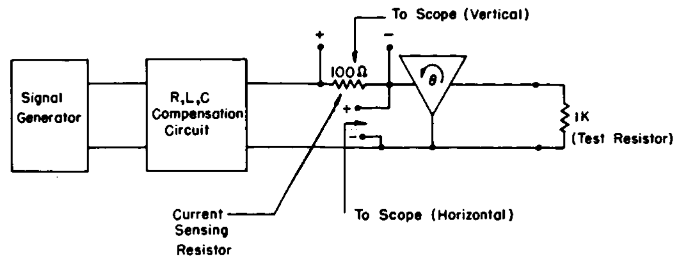

Fig. I
Measuring Circuit for Determining
Rotator Frequency Characteristics

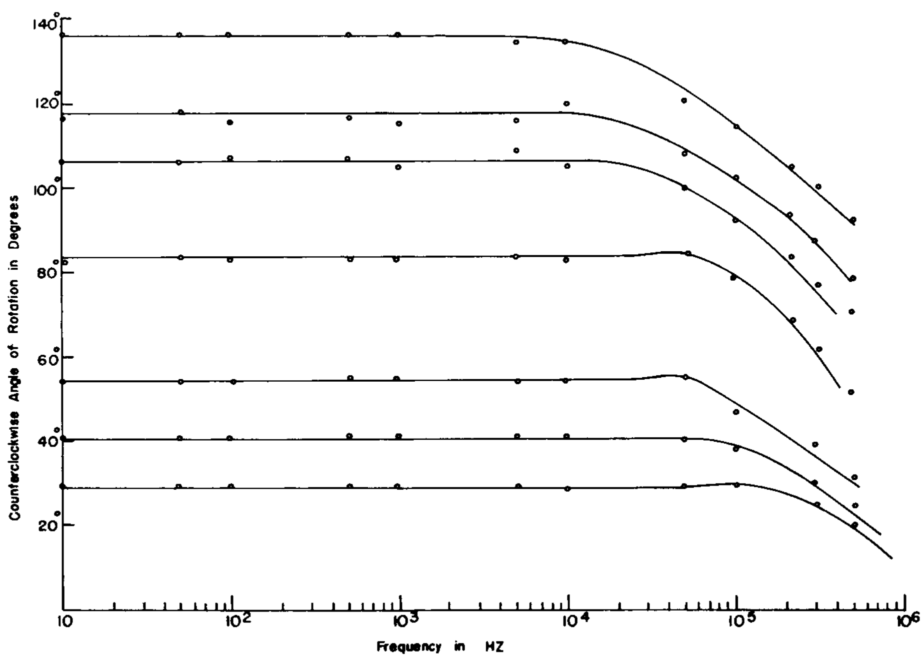

Fig. 2 FREQUENCY CHARACTERISTICS OF A TYPICAL R-ROTATOR
(REFERENCE TERMINATION- 1000 Ω)

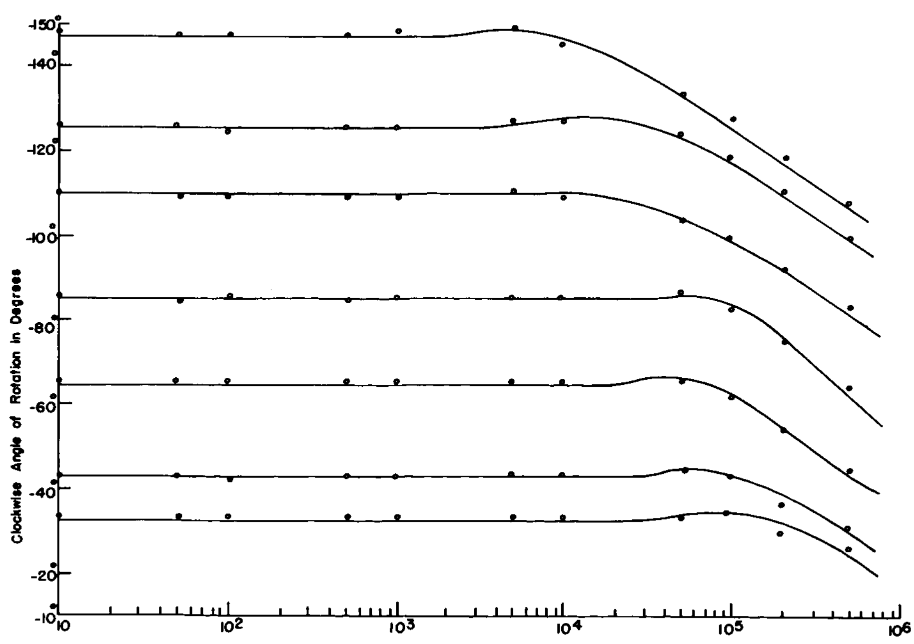

Fig. 3 FREQUENCY CHARACTERISTICS OF A TYPICAL-R ROTATOR
(REFERENCE TERMINATION- 1000 Ω)

From Figs. 2 and 3 it can be seen that the Rotator Frequency Characteristic is flat to approximately 10 KH_z, then rises slightly, and finally falls off around 100 KH_z. The above deterioration at high frequencies is clearly the result of the frequency characteristics of the transistors used in the Rotator. Therefore, to improve the frequency characteristic of the Rotator, higher frequency transistors must be used.

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D. IMPROVED HIGH POWER DC INIC FOR THE ROTATOR

R. Adams

L. O. Chua

The critical element in the new rotator[1] circuit, using either the "pi" or "tee" configuration, is the negative impedance branch. While there are many means available for obtaining negative resistance, but not so many for obtaining negative inductance or capacitance, it was decided that the most versatile way of achieving negative impedances for reasonable power levels was to use the negative impedance converter (NIC). One possible design has been previously described in the preceding Semi-Annual Report. This report presents the results of attempts to achieve a new design with improved performance.

The new circuit is shown in Figure 1, and the equivalent circuit model used for analysis in Figure 2. The improved performance of the circuit is due, in part, to the symmetrical arrangement. Analysis of the circuit using the model of Figure 2 yields, after some simplification:

i) $$g_{11} = \left(\frac{1}{R_6} + \frac{1}{r_{c_1}} + \frac{1}{r_{c_3}} \right) - g_{12} \left(\frac{1}{R} + \frac{1}{r_{c_2}} + \frac{1}{R_4} \right) + \frac{(1-\alpha_4)}{R_5}$$

ii) $$g_{12} = \frac{- \dfrac{(1-\alpha_3)}{(1-\alpha_4)} \left[1 + \dfrac{(1-\alpha_{12})}{re_2+R_2} \right] + \dfrac{\alpha_3 \alpha_1}{(1-\alpha_4)(1-\alpha_1)}}{1 + \left(\dfrac{re_1+R_1}{re_2+R_2} \right) \dfrac{\alpha_2 \alpha_5}{(1-\alpha_4)(1-\alpha_1)} + \dfrac{re_1+R_1}{R_3(1-\alpha_1)} + \dfrac{(1-\alpha_2)(re_1+R_6)}{(1-\alpha_1)(re_2+R_2)}}$$

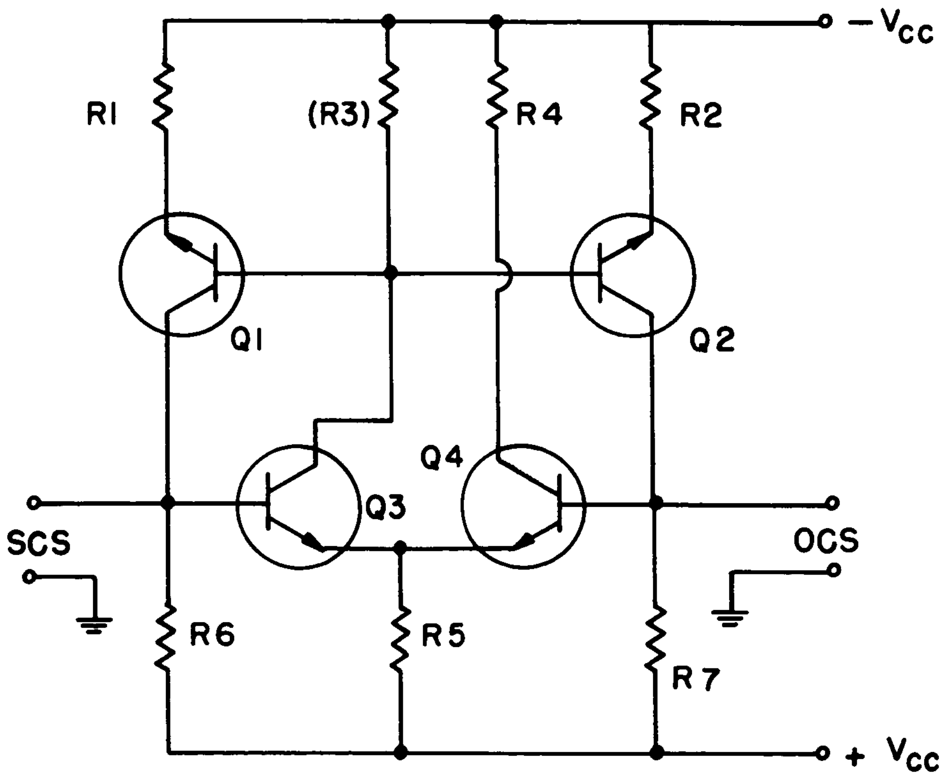

Figure 1.

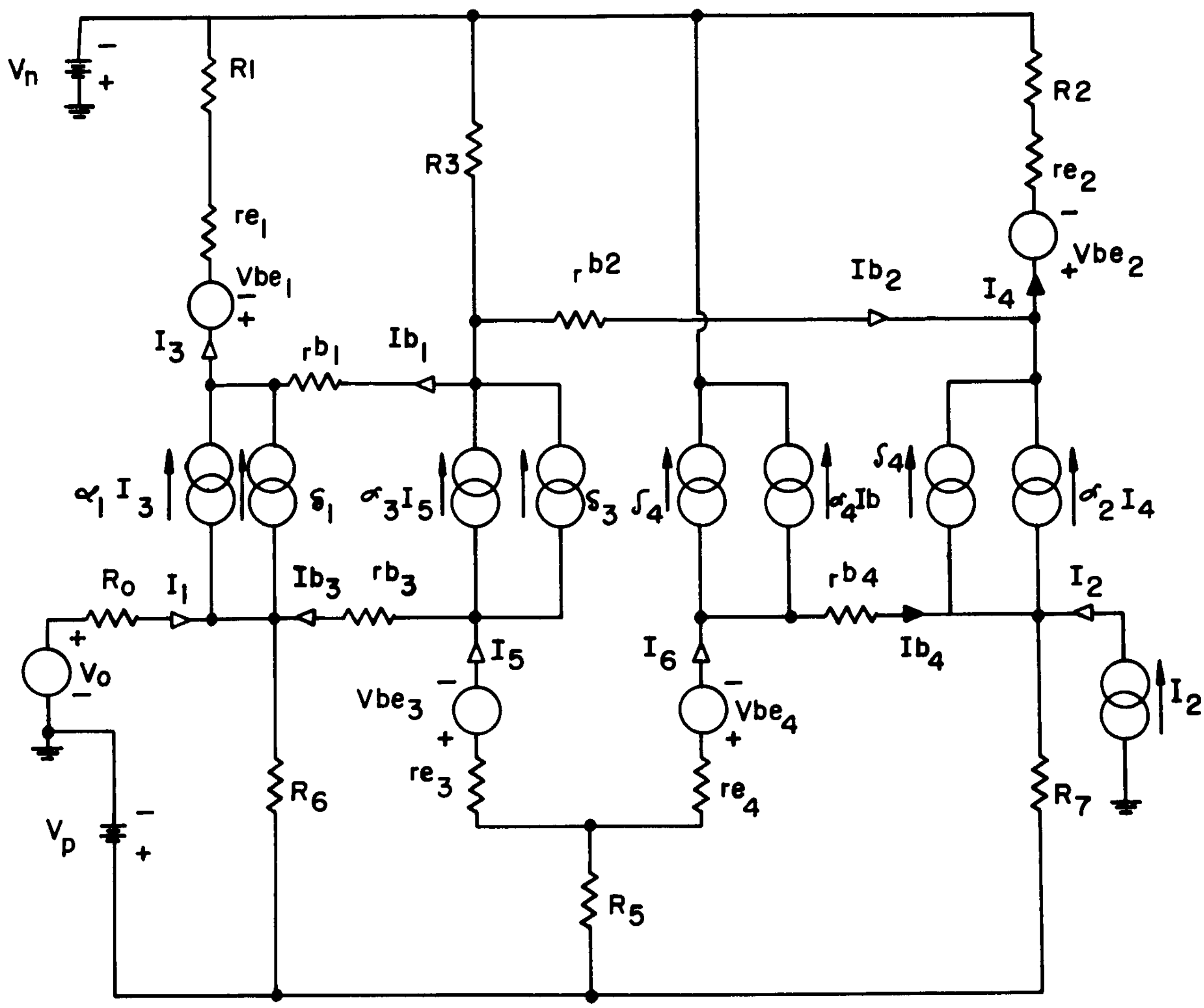

Figure 2

Equivalent Circuit Model for Figure 1.

$\delta = I_{co}$ current source

$\frac{1}{r_c}$ assumed negligible

iii) $g_{12} \doteq \dfrac{\alpha_1}{\alpha_2} \dfrac{(re_2+R_2)}{(re_1+R_1)}$

iv) $g_{21} \doteq 1$

v) $g_{22} = - g_{12} (1-\alpha_1) [rb_4(1-\alpha_4) + re_4+re_3+(1-\alpha_3)rb_3]$

and for DC sensitivity:

vi) $$I_1 = I_2 g_{12} + \frac{Vbe_2-Vbe_1}{(re_2+R_2)} + \frac{Vbe_4-Vbe_3}{R_7} g_{12}$$

$$+ [\delta_1-\delta_{12}g_{12}] + [\delta_3-\delta_4 g_{12}] + V_P \left[\frac{g_{12}}{R_7} - \frac{1}{R_6}\right]$$

From these expressions, the improvement in performance is seen to be derived from the high inherent feedback and the symmetry. For good AC performance R_3 should be large (e.g. infinite), the loop gain $\beta_1\beta_4$ large, R_1 and R_2 large, R6 and R7 equal, and g_{12} set to unity. Matching Q1 to Q2 and Q3 to Q4 will reduce g_{11}. A Darlington configuration for Q1 and Q2 will densitize g_{12} to $\alpha 1$ and $\alpha 2$. Im comparison, the Yanagisawa circuit shows little improvement over the inherent nature of the transistors, and relies upon using transistors with low r_b and V_e, high r_c, and stable alphas for good performance.

The DC performance of the circuit is also an improvement. In the Yanagisawa circuit the cancellation terms of vi) do not appear. With regard to equation vi), the dominant source of DC sensitivity is seen to be the Vbe_2-Vbe_1 term, as the I_{co} terms can be made neligible with the use of silicon transistors, and the (Vbe_4-Vbe_3) term is due to the low power input transistors, which can be matched thermally and electrically with comparative ease. Some desitization of the output transistors is possible with larger values of R , using matched transistors, and maintaining good thermal contact between the via a large heat sink. Also, changes in DC level can be caused by mismatching or unequal thermal heating in R6 and R7. The failure of this circuit to adequately desensitize the high power output stages is the major drawback of Figure 1.

A preactical implementation of Figure 1 is shown in Figure 3. R8 and R9 compensate for some parasitic instability near negative and positive saturation. Higher values of R1 and R2 will serve a similar purpose with a consequencial loss in voltage swing. Capacitor CI compensates for some stray capacitance across the open circuit stable (OCS) sides, primarily in the measuring equipment. Improvements in linearity, frequency response, and drift were noted over that of the Yanagisawa circuit previously reported, roughly by an order of magnitude or greater.

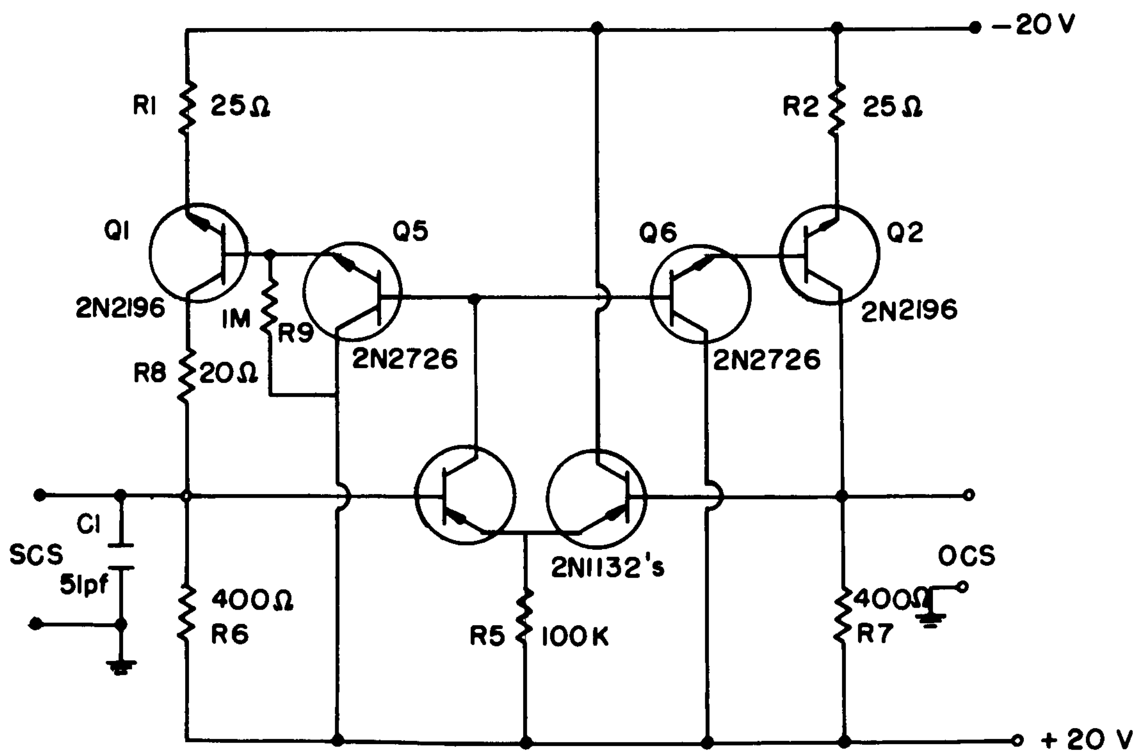

Figure 3.

III. PARAMETRIC AMPLIFIER STUDIES

A. A NEW METHOD FOR THE DETERMINATION OF GAIN IN PARAMETRIC AMPLIFIERS

J. V. Adams

D. R. Anderson

With the advent of the semiconductor diode, practical parametric devices have become a reality. For a little more than a decade, network theorists have been studying the fundamental characteristics of these devices. The usual procedure is to replace the diode by a time-varying capacitor whose capacitance as a function of time is sinusoidal. A linear time-invariant network containing such a capacitance may be modeled as shown in Figure 1 where $Y(\omega)$ is assumed to have a shunt capacitance $C_0 > 2$ so that the net capacitance is positive for all time. We define the impedance $Z_0(\omega)$ by the relation

$$V(t) = \sum_{k=-\infty}^{\infty} [Z_k(\omega) I e^{-j(\omega_s+k)t} + Z_k(\omega)^* I^* d^{j(\omega_s+k)t}]$$

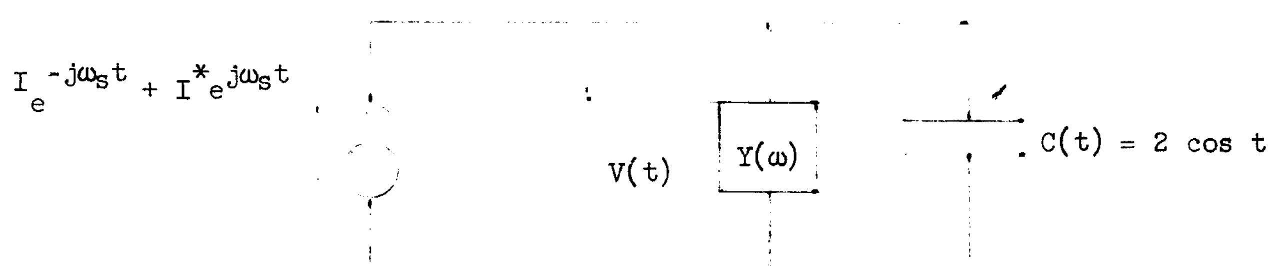

Fig. 1. Normalized Model for a Linear Parametric Amplifier

A problem of considerable importance in the design of a parametric amplifier is the determination of intervals of gain. Previous results based on the Harmonic Balance analysis are overly optimistic and are in some cases extremely inaccurate. This paper utilizes an analysis developed by Leon and Adams to derive a rigorous as well as simple algorithm for determining.

The algorithm is motivated by Leon and Adam's partial fraction expansion of the impedance $Z_0(\omega)$ of a parametric circuit relative to the fundamental of the output. A method of Perron is used to obtain the relative error for each of a convergent sequence of rational function approximations to $Z_0(\omega)$. The various rational function approximations are expressible in terms of $Y(j\omega)$, the admittance of the circuit seen looking back from the time-varying capacitor. Moreover, the approximations, together with the error estimates, yields a sequence of increasingly accurate refinements of the Harmonic Balance analysis. As a consequence, there results a family of simple and increasingly precise methods of determining intervals of gain.

B. PHASED LOCKED LOOP STUDIES

B. J. Leon L. L. Cleland

Three basic nonlinear problems associated with a phase locked loop (PLL) are under investigation; frequency acquisition or tracking without noise, phase error analysis with noise, and intermodulation distortion.

Viterbi[1] has shown that, when a second order PLL (without noise) is out of lock and in the process of pulling in, the approximate time required to lock is given by (1) $t \simeq \dfrac{\Omega_o^2}{K^2\alpha}$. Ω_o is the initial frequency error, K is the loop gain, and α is the loop filter, $G_1(s)$, parameter given in

$$G_1(s) = K_1 \left[1 + \frac{\alpha}{s}\right].$$

Also the approximate decrease in frequency error, y, per cycle of phase error, x, (for large y) is given by $\delta y \simeq \dfrac{\alpha\pi}{K[y^2(-\pi)-1]}$.

According to equations (1) and (2) the time required to lock is proportional to $\dfrac{1}{\alpha K^2}$ and the frequency decay per cycle is proportional to $\dfrac{\alpha}{K}$, thus increasing the rate of frequency decay per cycle and the time required to lock. This seemingly contradictory result can easily be explained. The time required to achieve lock is defined to be the time required for the system to stop skipping cycles and reach a terminating curve on the phase-plane plot. Increasing K decreases the number of cycles skipped, and thus the time required for frequency lock is decreased. Increasing α appears to yield still better results. Increasing α would tend to increase the rate of frequency decay per cycle but still decrease the time required for frequency lock. In fact, increasing both would be desirable. For example, multiplying both α and K by the same factor, ρ, does not change the loop damping factor, $\zeta = \frac{1}{2}\sqrt{\frac{K}{\alpha}}$, but increases the loop natural frequency, $\omega_n = \sqrt{K\alpha}$, by ρ. The plot shown in Figure 1 indicates this would increase the frequency lock range by ρ.[2,3,4] Since existing PLL optimum filter analysis specifies a particular (constant) α and K, this simple result has not been used to improve PLL performance.

Suppose K and, if possible, α are nonlinear functions of the phase or frequency error such that near lock (where optimum filter analyses are valid) the values of K and α are large. Then, when the system is out of lock and skipping cycles, the average values of K and α would be ρK and $\rho\alpha$, $\rho > 1$. Thus, improved performance would result.

Schilling[5] has shown for a second order system with any signal to noise ratio that during the locking process the system should be critically damped; however, after locking the damping factor should be reduced to decrease the variance of the phase jitter. This again suggests making the loop gain a nonlinear function of the phase or frequency error. Observe that the general type of nonlinearity needed is the same as the one suggested above.

Many PLL applications require both low threshold and a high signal to noise ratio. The high signal to noise requirement makes the intermodulation distortion (IMD) problem an important one, especially for multichannel signals. The existing PLL nonlinearities and/or the proposed additional nonlinearities would certainly cause IMD. This problem and the associated classical problem of defining bandwidth in a nonlinear system are under investigation.

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SECTION 6

MATERIALS

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N67-33614

I. MAGNETICS

A. A STUDY OF BALANCED MAGNETIC STRUCTURES IN A HIGH SPEED BIPOLAR SENSE AMPLIFIER*

F. J. Friedlaender

J. R. Eaton, Jr.

A balanced magnetic structure consisting of two non-linear magnetic toroids has been combined in a circuit with a linear capacitor to achieve a second-subharmonic buildup of capacitor voltage when pumped by a current pulse train. By utilizing the retentivity properties of the magnetic materials, the phase of the buildup can be controlled by application of a small steering current at any time prior to the pump sequence, and operation is achieved independent of the pump repetition rate.

A mathematical model of operation is being formulated, based upon experimental observation, which is providing insight into device operation, optimization and specification of required material properties.

It is anticipated that this study will provide the background for the development of high speed bipolar magnetic sense amplifiers for use with computer memory systems.

*This research was sponsored by U. S. Army, Navy, and Air Force in the Joint Services Electronics Program, Contract ONR N00016-66-C0076-A04

B. A MAGNETIC CORE ANALOG MEMORY*†

F. J. Friedlaender

J. D. McMillen**

1. Introduction

Flux reversal in ferromagnetic tape wound cores takes place mainly through the growth and motion of 180° domain walls. It has been suggested[1-3] that the particular domain geometries found during flux reversal depend upon the magnitude of the magnetic field which is applied to cause flux reversal, the initial remanent flux level, and the manner in which this flux level was achieved.

This research deals with the concept of controlled domain wall formation and motion in 50% nickel-iron tape cores; the control is effected by proper selection of low-field and high-field pulses, each of which is of insufficient duration to cause complete flux reversal. Low field pulses are less than $2H_c$ in amplitude, and high-field pulses are greater than $4H_c$, where H_c is the dc coercive field of a major hysteresis loop.

2. Bias Restoration of a Core Partially Switched by a High Field

It is possible to change the flux level of a non-saturated core with much less than the dc coercive field strength. For instance, if the initial flux level is set from positive saturation remanence to 25% below positive saturation remanence by a high-field pulse of approximately $4H_c$, the flux level may then be restored to a level close to positive saturation remanence by a field less than H_c.

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**Formerly with Purdue University, Lafayette, Indiana.
† INTERMAG, April, 1967.

Actually, as long as the initial flux level is set by a high-field pulse, there is a wide range of possible levels to which the core may be set. The only difference in restoration is that the further away from positive saturation, the longer time is needed for restoration with any given bias field. For 25% flux change, the restoring time corresponds to switching times of saturated cores caused by high-field pulses.

Teig and Kiseda make use of such a property in ferrite and 1/8 mil tape cores in their "torroidal non-destructive read memory element using bias restoration."[4] The operation of this device relies on the reduced width of a minor hysteresis loop along with the above idea of high-field pulses followed by a sub-coercive bias field. By sensing the voltage across a winding of the core during the high-field pulse, it is possible to determine whether the core has been set to negative or positive saturation before the high field pulse is applied. The sub-coercive bias field restores the original flux level and the sequence of high field pulse followed by sub-coercive bias may be repeated indefinitely without permanently changing the flux level of the core.

3. Operation of Magnetic Core Analog Memory

It has been shown[3,5] that when a high-field pulse is applied to a core partially switched by a low field, the magnitude of the voltage response waveform is determined by the initial flux level; for instance, the peak of the voltage waveform is related to initial flux level. Also, it has been shown above that if a high-field pulse partially switches a core, the original flux level can be reset by a sub-coercive bias field. In addition to this, it is found that if an intermediate flux level is established in a core by means of a low-field pulse, then the application of a short duration high-field pulse followed by a sub-coercive bias results in the restoration of the intermediate flux level which existed just prior to the high-field pulse.

These facts make it possible to establish any arbitrary flux level in a tape core, obtain a signal pulse whose peak amplitude is related uniquely to the flux level, and restore the flux level. An "analog memory" has been built to operate in this manner; the output device is a dc milliammeter which is given from the amplified output of a peak detector. It has been found that even as many as 100,000 repetitions of the high-field pulse fail to change the output displayed by the meter.

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5. Woodbury, R.C., "Nondestructive Readout and Partial Switching of the Net Magnetization in 50 Percent Nickel-Iron", SET Technical Report No. 4651-3, April, 1965.

C. ANHYSTERETIC MAGNETIZATION OF NI-FE TAPE CORES*

F. J. Friedlaender

J. D. McMillen

An experimental investigation of the anhysteretic magnetization of "square-loop" Ni-Fe tape cores indicates that anhysteretic remanence and magnetization curves are identical below saturation remanence and can be reproduced with an accuracy of about ± 10%. The initial slope of the anhysteretic magnetization does not depend on core geometry for a wide range of sizes, and takes on values from 3.6 to 29 henry/meter for different 50% Ni-Fe and square permalloy cores. By comparing hysteresis loops taken from an initial anhysteretic state, with minor loops taken conventionally, the anhysteretic state seems to correspond to a state of about equal average flux at each cross-section of the tape. On the other hand, saturation in opposite directions for the inner and outer wraps of the tape seems to occur when partial flux reversal is obtained by slowly increasing an applied dc field.

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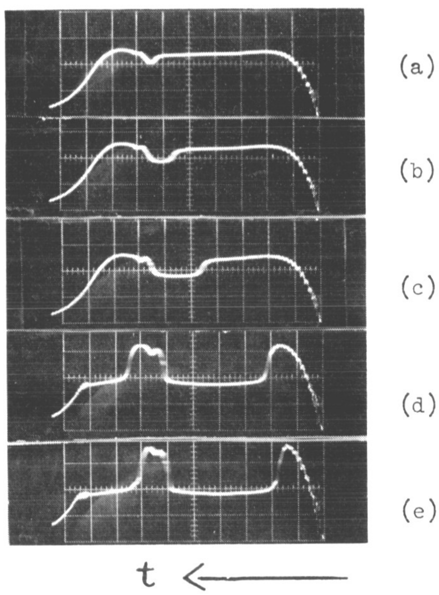
*This research was sponsored by U.S. Army, Navy, and Air Force in the Joint Services Electronics Program, Contract ONR N00016-66-C0076-A04.

D. HIGH FIELD INSTABILITIES IN CADIMUM SULFIDE

I. C. Chang

D. S. Zoroglu



Vertical Scale: 100 Amp/div-cm^2

Horizontal Scale: 0.5μsec/div.

Temperature: 300°K

(a) E_a = 1070 volts/cm

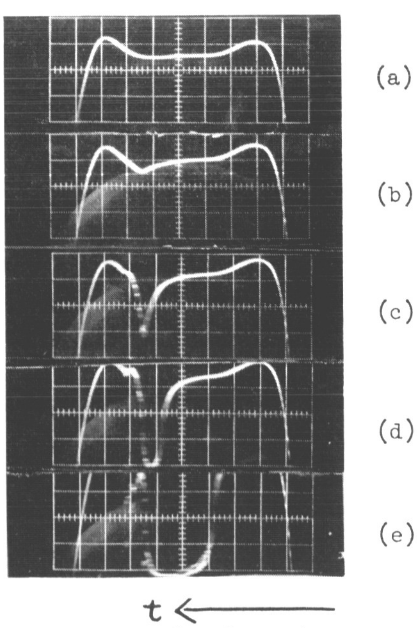
(b) E_a = 1088 volts/cm

(c) E_a = 1140 volts/cm

(d) E_a = 1468 volts/cm

(e) E_a = 1720 volts/cm

Fig. 1. The development of current waveforms for a sample at room temperature.



Vertical Scale: Not calibrated.

Horizontal Scale: 0.5μsec/div.

Temperature: 77°K

(a) Applied voltage pulse

(b) Current waveform with $E_a = 450\ \frac{V}{cm}$

(c) Current waveform with $E_a = 470\ \frac{V}{cm}$

(d) Current waveform with $E_a = 480\ \frac{V}{cm}$

(e) Current waveform with $E_a = 560\ \frac{V}{cm}$

Fig. 2. Current waveforms for the sample as in Fig. 1 at 77°K.

D. HIGH FIELD INSTABILITIES IN CADIMUM SULFIDE

I. C. Chang

D. S. Zoroglu

During this semester we continued our work on the acousto-electric instability in semi-conducting Cds [1]. More experimental results were obtained, and a phenomenologic model was propared.

Experiment: All the samples used in our experiments were cut in the shape of a bar (1mm x 1mm x 5mm) with c-axis perpendicular to the long side. Constant voltage pulses of duration 4μsec to 8μsec were applied across the sample. The development of current waveforms for a sample at room temperature are shown in Fig. 1, for a series of constant voltage pulses. The corresponding current waveforms for a sample at 77°K are shown in Fig. 2. The results are similar for the two temperatures except that the oscillation threshold decreases as the sample is cooled down. In our experiments the threshold electric field is 1040 v/cm at 300°K and 450 v/cm at 77°K.

The formation, propagation, and amplification of the "acousto-electric field packet" or domain were studied by a pair of electrical probes. The experimental setup for this "probing" is shown in Fig. 3. Since the reading of probe is

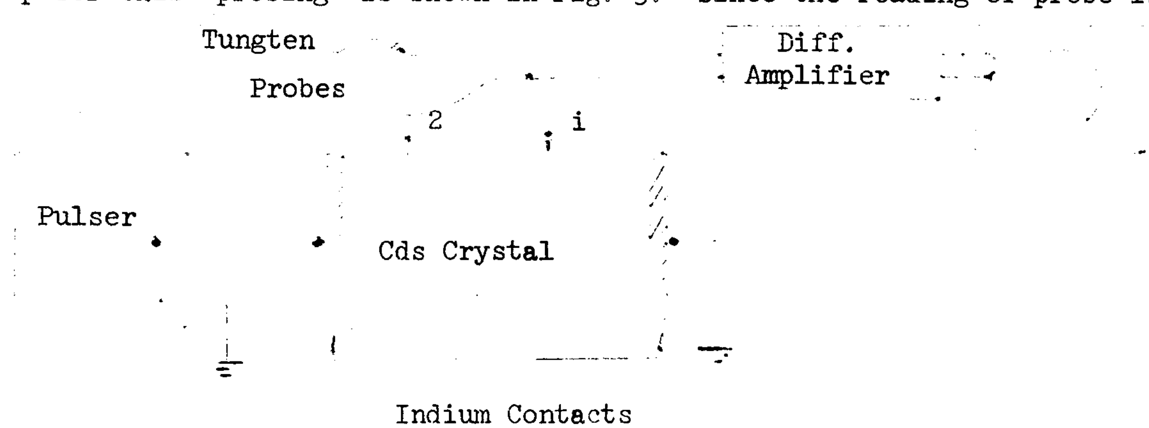
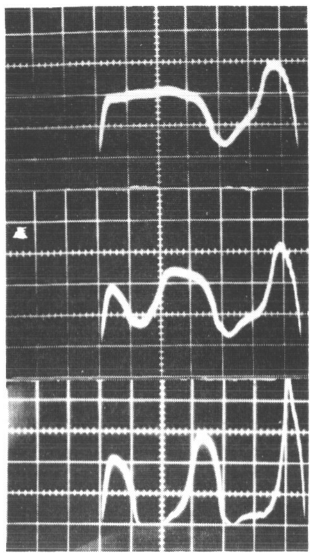


Fig. 3. The experimental setup for "probing."



(a)

(b)

(c)

Vertical Scale: Not calibrated

Horizontal Scale: 1μsec/div.

Temperature: 300^{o}K

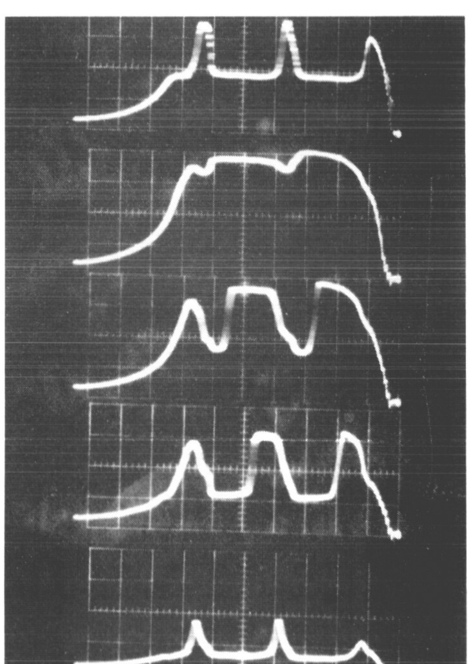
(a) E_a = 1450 volts/cm

(b) E_a = 1500 volts/cm

(c) E_a = 1650 volts/cm

Notice that the domain becomes more peaked and narrower as the applied field increases

Fig. 4. The probe output in the set-up shown in Fig. 3.



(a) Current through the sample
Vertical Scale: 1.06 Amp/div.
Horizontal Scale: 1μsec/div.

(b) Probe output at 0.1 mm from anode
Vertical Scale: 200 volts/div.
Horizontal Scale: 1μsec/div.

(c) Probe output at 1.2 mm from anode
Vert. and Hor. Scales are the same

(d) Probe output at 2.7 mm from anode
Vert. and Hor. Scales are the same

(e) Probe output at 4.0 mm from anode
Vert. and Hor. Scales are the same.

Fig. 5. Probe output with probe 1 grounded and probe 2 as indicated above. Picture (a) in the current waveform. For all pictures E_a = 1650 volts/cm

proportional to the area under the electric field distribution curve between the two probes, a time display of the probe output therefore indicates the growth pattern of the domain. The probe output for a series of electric fields are shown in Fig. 4. The probe output for a number of probe positions are shown in Fig. 5.

We have also tried the experiments of microwave emission from Cds. We placed the sample in a S-band waveguide that was connected to a microwave super-hetrodyne system. The sensitivity of the detecting system was about -80dbm. We did not observe any microwave emission either at $300^{\circ}K$ or at $77^{\circ}K$.

Theory: It is clear that the current oscillation we observed in Cds is a consequence of the generation, propagation, and amplification of the acousto-electric domains. A complete understanding of the generation process of domains is still not available.[1,2] We proposed, however, a phenomenologic model that fully described the motion (i.e., propagation, amplification, and saturation) of the domains. To illustrate our model, we depict the electric field distribution in Fig. 6. As the applied electric field is greater than

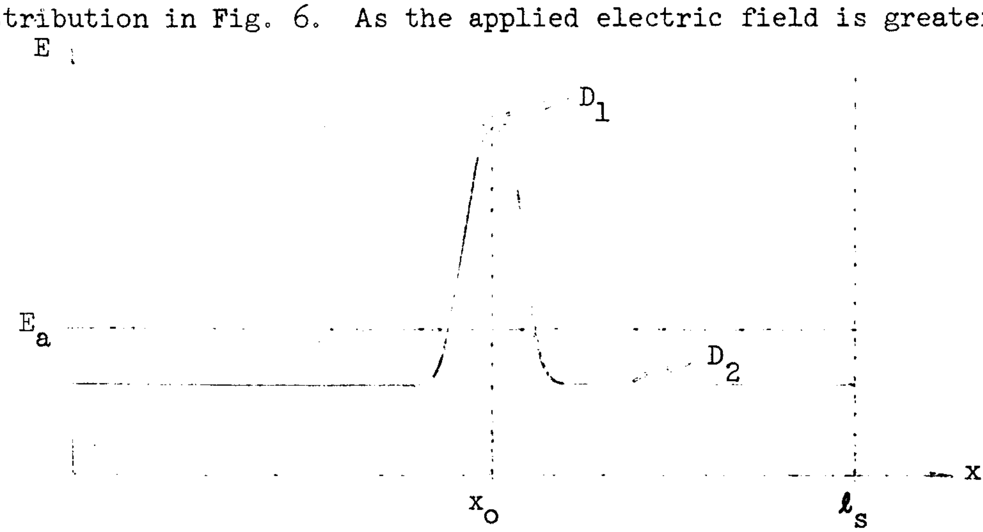


Fig. 6. The electric field distribution in the sample for some $E_a > E_t$, at time $t_o = V_s^{-1} x_o$ where V_s is the velocity of acoustic propagation.

E_t, the threshold for amplification, because of interaction of acoustic flux and drifts electrons, the domain height increases exponentially as it travels towards the anode. The electric field in the rest of the sample decreases so that the total voltage across the sample is conserved. At some distance from the cathode, nonlinear mechanism sets in that saturates the domain. Referring to Fig. 6, we have by small signal analysis [1]

$$D_1(x) = E_a \exp \alpha x \approx E_a \exp \alpha_o \gamma x \tag{1}$$

where E_a is the applied electric field, $D_1(x)$ is the height of the domain, α is the acoustic gain of the sample, and γ is the multiplication factor given by

$$\gamma = \frac{vd}{vs} - 1 = \frac{D_2}{E_t} - 1 \tag{2}$$

where: $E_t = \frac{v_s}{\mu}$ = threshold drift field for acoustic amplification

D_2 = bulk electric field everywhere except at the domain.

For constant voltage operation the area under the field distribution is conserved; i.e.

$$\int_o^{\ell_s} E dx = E_a \ell_s \tag{3}$$

where ℓ_s is the length of the sample. In our model we assume that the domain shape is triangular, Eq. (3) reduces to

$$\frac{W}{2} D_1 + D_2(\ell_s - \frac{W}{2}) = E_a \ell_s \tag{4}$$

where W is the "width" of the domain. From Eq. (1) and (2) we have

$$D_1 = E_a \exp \alpha_o(\frac{D_2}{E_t} - 1) x \tag{5}$$

The coupled equations (Eq. (4) and Eq. (5)) can be solved for D_1 and D_2 as a function of x. The plot of D_1 and D_2 is given in Fig. 7. It is seen that qualitative agreement with experiment is obtained.

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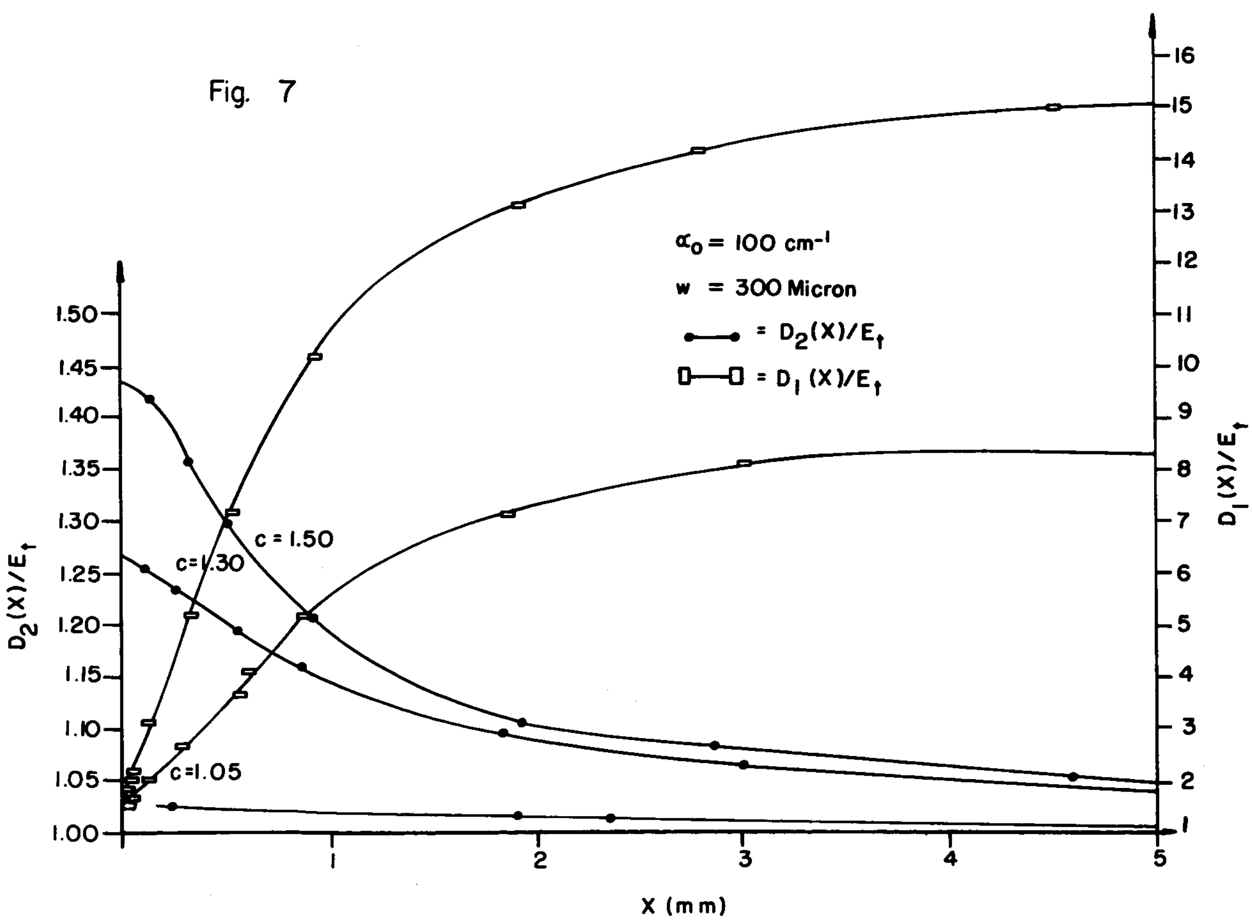
Fig. 7



N67-33615

II. SEMI-CONDUCTORS

A. AN INVESTIGATION OF THE HETEROJUNCTION INTERFACE*

H. W. Thompson, Jr.

A. L. Reenstra

A profile of the mixing of the silicon and germanium in an alloyed Ge-Si heterojunction was determined for the alloyed heterojunction by employing an electron microprobe. This measurement determined the mixing of the silicon and germanium at the junction to be controlled by diffusion. A master plot shown in Fig. 1 of percent germanium versus time normalized distance was made for a series of samples for which the time allowed for interdiffusion was varied. The master plot showed the samples to be time dependent and follow closely a complementary error function profile. From the coincidence of the profiles the process controlling mixing was concluded to be diffusion with a diffusion constant of 1.4×10^{-11} um^2/sec. The diffusion constant was also shown to be independent of temperature over the range of interest - 941°C to 971°C.

In n-n type heterojunctions, a p-type layer is observed by measuring the V-I characteristics. Indium balls, alloyed in a hydrogen atmosphere into the germanium side of the heterojunctions at the interface, serve as current and potential contacts to the interface p-layer. By this method it was possible to measure the conductivity and Hall coefficient of the interface region over a temperature range of 270°k to 77°k without interference from the bulk material. Capacitance of the p-n junction to the bulk germanium material was measured to determine the carrier concentration in the p-layer and, in conjunction with the Hall coefficient, the width of the p-layer. These measurements were made on samples for which the interdiffusion time, recrystallization rate

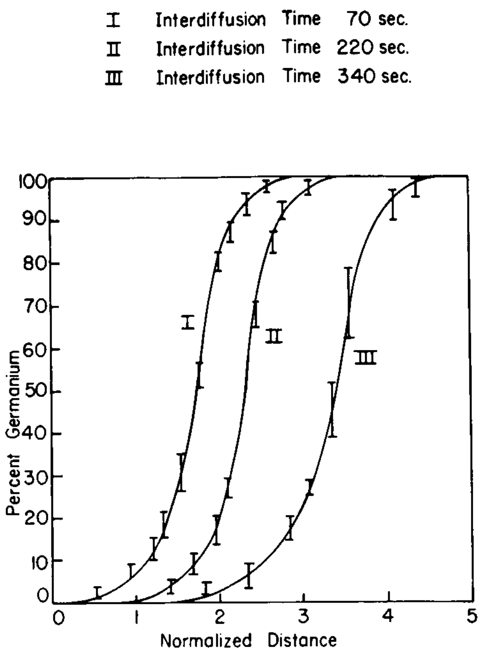
*This work was sponsored by U.S. Army, Navy, and Air Force in the Joint Services Electronics Program, Contract ONR N00016-66-C0076-A04.

of the germanium, and the impurity of the bulk material had been varied.

Since the width of the p-layer at the interface is not known, a reduced carrier concentration and a reduced conductivity, which is the carrier concentration and the conductivity multiplied by a constant -- the p-layer width -- could only be determined. The reduced carrier concentration, reduced conductivity, mobility of the carriers, and carrier concentration versus the inverse of the absolute temperature are shown in Figs. 2, 3, 4, and 5 respectively for samples in Table 1.

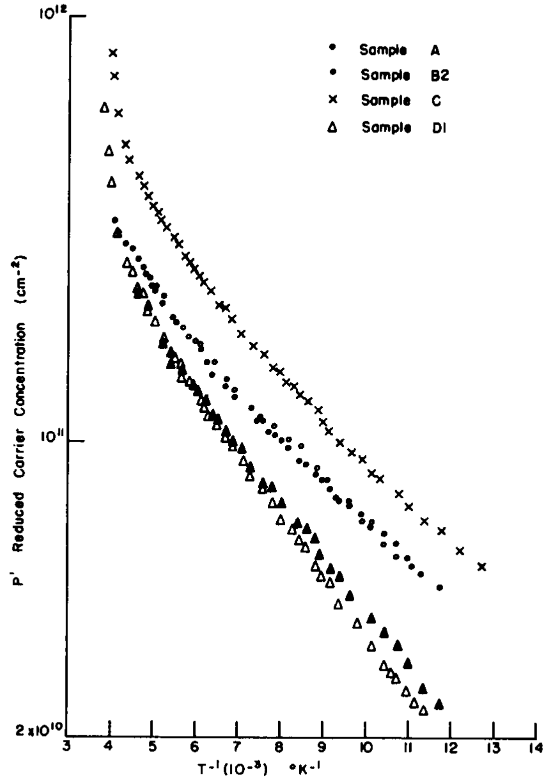
The reduced carrier concentration and reduced conductivity curves demonstrate the isolation of the p-layer from the bulk material. At the very highest temperatures the curve shows a sharp break. This shows the effect of leakage current through the bulk material on the parameters, and the rapid rate at which the leakage current decreases with temperature. Therefore, the p-n junction can be concluded to support good isolation of the p-layer over most of the temperature range: These straight line segments are apparent in the reduced carrier concentration curve in Fig. 2. These give ionization energys of 0.027, 0.019, and 0.017ev which are not identifiable with any of the known impurity levels in either germanium or silicon. Therefore, it must be concluded that the energy levels are caused by dislocations or impurities in the alloyed region.

Indirectly the p-layer width was measured from the electrical measurements. By dividing the reduced carriers concentration by the carrier concentration, the width of the p-layer was determined. The width of the p-layer is shown in Table 2 for the samples in Table 1.



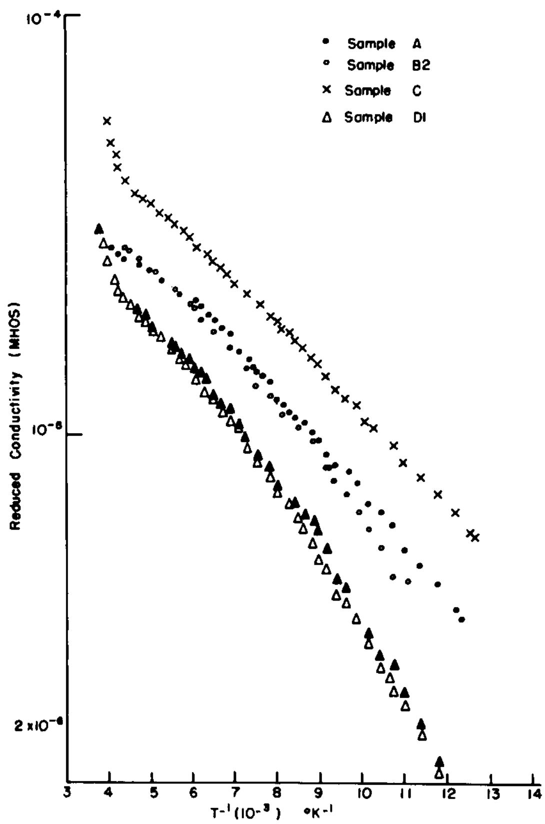
GERMANIUM CONCENTRATION PROFILE

FIG. I.



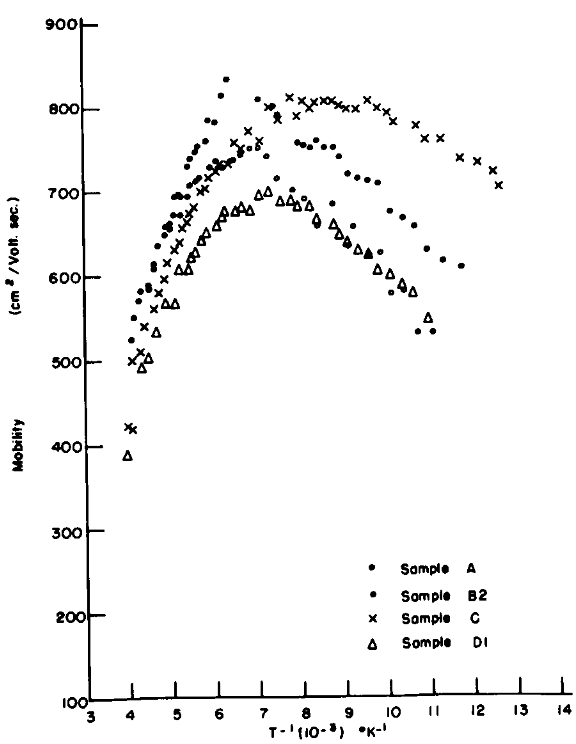
Reduced Carrier Concentration vs Inverse Temperature

Fig. 2



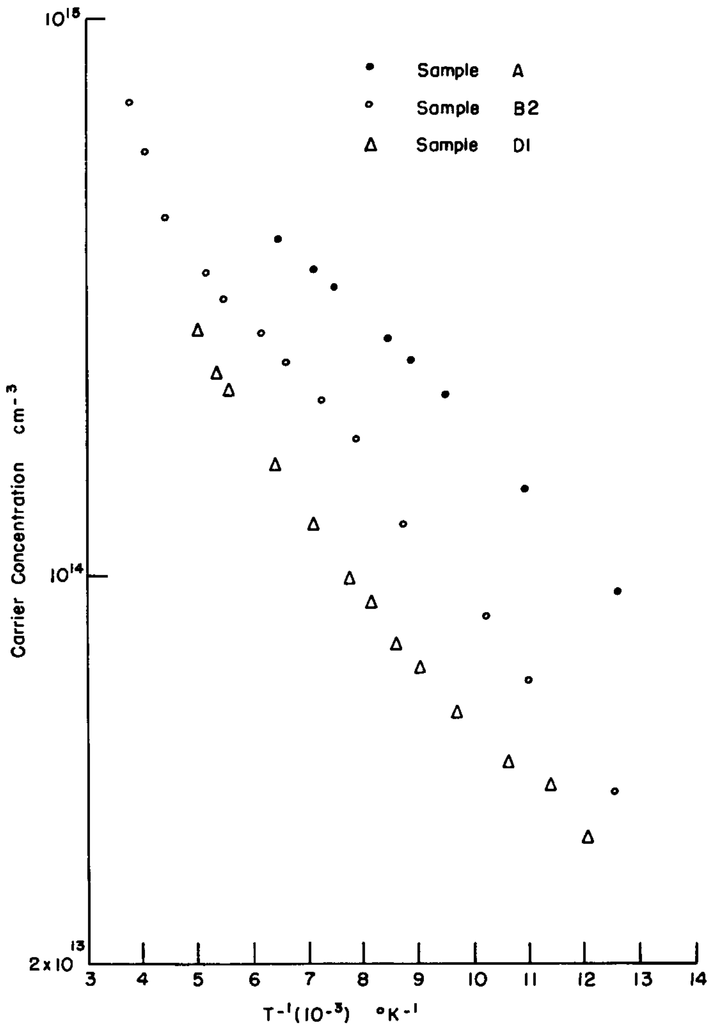
Reduced Conductivity vs Inverse Temperature

Fig. 3



Mobility vs Inverse Temperature

Fig. 4



Carrier Concentration vs Inverse Temperature

Fig. 5

Table 1.

Electrical Measurement Samples

Sample	Interdiffusion Time	Recrystalization Cooling Rate	Starting Material	
			Silicon	Germanium
A	70 sec.	0.76°C/sec.	N-type 100 ohm cm	0.25 ohm cm arsenic doped N-type
B1	340 sec.	0.76°C/sec.	N-type 100 ohm cm	0.25 ohm cm arsenic doped N-type
B2	340 sec.	0.76°C/sec.	N-type 100 ohm cm	0.25 ohm cm arsenic doped N-type
C	70 sec.	0.76°C/sec.	N-type 100 ohm cm	0.16 ohm cm antimony doped N-type
D1	70 sec.	0.20°C/sec.	N-type 100 ohm cm	0.25 ohm cm arsenic doped N-type
D2	70 sec.	0.20°C/sec.	N-type 100 ohm cm	0.25 ohm cm arsenic doped N-type

Table 2

Sample	Width d Microns	
	T = 100°K	T = 166°K
A	3.42	3.78
B2	6.5	6.1
D1	6.0	6.2

From the electrical measurements and the electron microprobe analysis, there is very strong evidence to show the energy levels in the energy gap at the interface to be caused by dislocations. The direct evidence of dislocations is the decrease in reduced carrier concentration with a decrease in the recrystallization rate.

Measurements rule out the two sources of impurities as the cause of the interface states, and show direct evidence of dislocations causing the interface states. It can therefore be concluded that the source of interface states in the alloyed germanium-silicon heterojunction is dislocations rather than impurities.

B. THE MEASUREMENT OF NOISE ON GERMANIUM-SILICON HETEROJUNCTIONS*

H. W. Thompson, Jr.

Gerold W. Neudeck

The noise of a Germanium-Silicon Heterojunction was measured at frequencies of 300 MH_z, 400 MH_z and 500 MH_z, and compared with the noise measured at the same frequencies in a high-conductance silicon diffused diode and a commercial

*This research was sponsored by U.S. Army, Navy, and Air Force in the Joint Services Electronics Program, Contract ONR N00016-66-C0076-A04.

hot carrier diode. The measurement of noise on all devices was obtained for several values of forward bias at each frequency.

The measurements at 300 MH_z and 500 MH_z indicate that the Ge-Si heterojunction has a noise figure that is lower than that of a silicon diffused diode, although the noise figure is still somewhat higher than that of the commercial hot carrier diode. The data at 400 MH_z and 500 MH_z was not obtained under ideal conditions (i.e., when other equipment was not in operation) and cannot be interpreted as being conclusive. The data suggests superiority of the heterojunction relative to the diffused device even as frequency is increased. However, further definitive work must be carried out at the higher frequencies.

This preliminary investigation indicates that the heterojunction is a low noise device. Further studies will be made to ascertain the best operating conditions and noise as a function of frequency.

C. BISTABLE SWITCHING NIOBIUM OXIDE DIODER

R. J. Schwartz

H. Luginbuhl

T. L. Chiou

Most of the effort during this reporting period has gone into obtaining very small area contacts and reproducible characteristics. While progress has been made in the former area through the use of photolithographic technique, this stability problem is still with us.

The voltage dependence of the current as a function of temperature has been measured for a number of units and was found to vary from unit to unit. Typical functional dependence are $I \alpha V^n$ where n is between 1 and 3.

D. DEVELOPMENT OF A HIGH FREQUENCY MOS FIELD EFFECT TRANSISTOR*

R. J. Schwartz

R. C. Dockerty

There is much interest in improving the high grequency performance of field effect transistors. For two reasons, the use of III-V compounds in the FET structure seems promising for higher frequency operation. The gain band-width product of a FET is directly proportional to the carrier mobility. Also, the short minority carrier lifetime, which limits the use of most III-V compounds in bipolar transistors, does not affect the operation of a FET, which is a majority carrier device. We are attempting to fabricate an MOS device using indium arsenide as the semiconductor.

A major problem in fabricating an FET is reduction of the surface state density at the oxide semiconductor interface to 5×10^{12} states/$(cm)^2$ or less. The surface state density can be determined from MOS capacitance measurements. Apparatus to fabricate MOS capacitors by deposition of silicon dioxide on InAs single crystals is being assembled.

E. AN INVESTIGATION OF THE STEP-RECOVERY DIODE**

H. W. Thompson

W. E. Drobish

The step-recovery diode is a p-n junction device whose reverse recovery current approximates a step function. The step-recovery diode is fabricated with a retarding field in order to enhance charge storage and constrain the minority carriers to the junction.(1) Thus, during the reverse transient, essentially all the stored charge is removed from the bulk before the carriers

* Supported by the U.S. Army, Navy, and Air Force in the Joint Electronics Service Program, Contract ONR N00016-66-C0076-A04.
** Supported by NASA, NGR 15-005-021.

at the junction are depleted. This results in a relatively long reverse conduction phase followed by an abrupt (usually less than one nanosecond) transition to cutoff. Because of its abrupt turn-off characteristics, the step-recovery diode is used for fast pulse-shaping applications, and pulse generation. The combination of its power handling capabilities (up to one watt at one gigahertz) and its switching speed is responsible for its wide use as a simple and efficient generation of harmonies at microwave frequencies.[2]

The objects of this study are to investigate the theoretical explanation of the behavior of the step-recovery diode, to experimentally verify the results and to predict optimization parameters. The static V-I characteristic, transient response, and capacitance variation of commercially available step-recovery diodes will be compared to the theoretical predictions. The preliminary literature survey has been completed and the commercially produced step-recovery diodes have been requested.

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III. SOLID STATE ENERGY CONVERSION

A. A P-I-N PHOTOVOLTAIC ENERGY CONVERTER

R. J. Schwartz

C. W. Kim

N67-33616

The p-i-n photovoltaic converter which was described in the previous semi-annual report has been fabricated and tested under high intensity illumination.

The following impurities were used to dope into the intrinsic germanium:

N-type	Sn-Sb
N-type	Au-Sb
P-type	Al
P-type	In-Ga
P-type	In-Al
P-type	Au-Ga

It was found that Al for p-type and Au-Sb for n-type resulted in the best electrical characteristics.

Open-circuit voltages of the devices were found to be 0.23 volts at about 7 watts/cm^2 of illumination from a tungsten lamp. This is well below the value of .5 volts which is to be expected for this device. The reason for this reduction is now being sought.

IV. THIN FILMS

A. FAILURE MECHANISMS IN THIN-FILM RESISTORS*

N67-33617

H. W. Thompson, Jr.

R. F. Bennett

Thin film cermet resistors on glass substrates have been tested to determine failure modes when operated at full rated power in high-temperature environment. Twenty substrates were fabricated by NAFI for our tests. These substrates utilized identical resistor-deposition methods but had termination structures fabricated by (1) chrome-copper over the cermet resistor film, (2) chrome under cermet plus chrome-copper over cermet, and (3) chrome-gold over cermet. Lead connection methods included (1) welding to termination pads at a position away from the cermet, (2) welding to termination directly above the cermet, (3) localized soldering to termination pad, and (4) soldering to completely-tinned termination pad. Effects of a protective coating over the terminations was also studied.

Two test groups were used to provide failure data. Test time for the first group was 1500 hours at 125°C; the second group had test times of 480 hours at 150°C and 670 hours at 175°C.

A total of 28 failures was experienced in the group of 144 resistors on test. These include 17 catastrophic failures in which the resistance changed by 100% or more, 8 drift failures in which the resistance changed by 10-100%, and 3 slow-drift failures with resistance changes in the 4-10% range. Approximately 25% of the failures arise from having soldered leads to the chrome-gold termination pads; this process removes a significant amount of gold from the

*This work supported under contract N0016366C0096 with NAFI.

termination and leaves an electrical and mechanical contact of questionable quality.

Data show that the method of fabricating the termination pad is not particularly important, but that the method of lead attachment is the most significant factor contributing to device failure. The soldered leads show insignificant failure rates (except on the gold termination where failure is attributed to other causes) when compared to welded lead terminations. Additionally, the protective coating of Sylgard 182 seems to provide significant protection.

Analysis of a failed termination by an ARL electron microprobe confirms a high oxygen content on the termination pads, and suggests oxidation as the chief cause of resistor failure. By probing the cermet element of failed resistors, it was shown that the resistive element is not responsible for the failures observed in this work.

B. A PRELIMINARY STUDY OF THE FAILURE MECHANISMS OF CdSe THIN FILM TRANSISTORS

R. J. Schwartz

R. C. Dockerty

A preliminary investigation into the failure mechanisms of CdSe TFT's was made. Several types of failures were observed. The transconductance, gate capacitance, and gate resistance were measured at various temperatures and frequencies. Both an initial transient and a long term drift were observed for these quantities. The response to a step input to the gate was measured and was found to be both undesirably slow and asymmetric. Possible mechanisms for the instabilities have been postulated.

N67-33618

V. QUANTUM ELECTRONICS AND MILLIMETER WAVES

A. RESONANT MIXING AND PARAMETRIC OSCILLATIONS AT OPTIMAL FREQUENCIES

I. C. Chang

J. C. Stover

The objective of the project is to investigate the enhancement of optical nonlinear process (mixing, harmonics generation, parametric oscillation) by the use of resonant optical structures.

A theoretical study of resonant optical process based on Lamb's self-consistency equations [1] has begun. The formulation is summarized as follows:

We start with the inhomogeneous wave equation assuming only electric dipole moment[1]

$$\frac{\partial^2 \overline{E}}{\partial t^2} - \frac{1}{\mu_o \epsilon}\frac{\partial^2 \overline{E}}{\partial t^2} + \frac{\sigma}{\epsilon}\frac{\partial \overline{E}}{\partial t} = -\frac{1}{\epsilon}\frac{\partial^2 \overline{P}}{\partial t^2} \tag{1}$$

The polarization P(Z,t) can be written as a finite sum of Fourier components [2]

$$\overline{P}(Z,t) = \Sigma\, P_n e^{j(\omega_n t-\beta_n Z)} + \tilde{P}_n e^{j(\omega_n t+\beta_n Z)}$$

$$+ \text{ complex conjugate} \tag{2}$$

The electric field in the structure can be decomposed into the normal modes in a similar manner. The equation of motion for the normal modes can be shown to be [2]

$$\frac{\partial E_n}{\partial Z} \pm \frac{k_n}{\omega_n}\frac{\partial E_n}{\partial t} + \alpha_n E_n + j(k_n-\beta_n)\, E_n = \frac{k_n}{2j\epsilon} P_n \tag{3}$$

where $k_n=\omega_n\sqrt{\mu_o\epsilon}$="Zeroth order" free space wave number

$\alpha_n = \frac{\sigma k_n}{2\omega_n\epsilon}$ = effective photon loss coefficient

The polarization term $P(Z,t)$ is the sum of contributions from the active laser media and from the nonlinear effect from the perturbing crystal.

$$P(Z,t) = P_a + \bar{\bar{X}}\bar{E}\bar{E} + \ldots + \text{higher order terms} \tag{4}$$

The resulting coupled wave equations [2] take different forms for different optimal structures and different nonlinear processes.

Our experimental program is at initial stage. The experiment of SHG using KDP as the nonlinear material is in progress [3]. The ruby laser is Q-switched by (a) Schott glass and (b) saturable dye (cryptocyanine in an isopropanel solution). Laser pulses of extreme short duration ($\sim$ 40 nsec) were obtained. The estimated power exceeded 1 magawatt.

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SECTION 7

LABORATORY FOR AGRICULTURAL REMOTE SENSING

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I. PATTERN RECOGNITION

A. DIMENSIONALITY REDUCTION

Various data reduction schemes are currently being applied to spectrophotometer data obtained by reflectance measurements on the Beckman DK-2 spectrophotometer.

Data is obtained at the highest sampling rate now available in the 0.5μ to 2.6μ range. Due to the number of sample points per curve, analytic and statistical studies are cumbersome.

Orthogonal data fitting schemes (principally orthogonal polynomials) are being applied to the data.[1] The results are to be applied to pattern recognition techniques as well as statistical studies.

By means of orthogonal polynomials, data curves with m points ($m = 211$) have been reduced to K points ($K \doteq 30$) with low mean square error.

Current investigation includes pattern classification performance before and after data reduction.

The polynomials are pointwise orthogonal, and are generated iteratively in a simple fashion. The scheme requires a much reduced computer memory, and facilitates large sample analysis.

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B. PATTERN CLASSIFICATION

A statistical recognition procedure has been programmed for use in crop classification studies. The procedure is based on a maximum likelihood decision criterion employing a multivariate Gaussian assumption on the distribution of features for each pattern class. The inputs to the program are estimates of the vector of means and the covariance matrix for each pattern class, and the samples to be classified. The computer implemented procedure classified the sample, printing out the probability that the sample is from each particular class. In addition, the fact that the true classification of presently available samples is known allows the computation of percent correct recognition. Using this program, some preliminary studies have been conducted, and the results are presented in Table I. For these studies 9 features (frequency bands) were employed to separate 3 classes (oats, wheat, and corn). The mean vector and covariance matrices were learned using between 100 and 200 samples for each class. The samples classified were the same samples used to estimate the multivariate Gaussian distribution.

Table 1. Results of Statistical Approach Using 9 Features and Multivariate Gaussian Assumption

True Class	Total No. of Samples	No. of Samples Classified as			% Correct Recognition
		Oats	Corn	Wheat	
Oats	164	163	1	0	99.4
Corn	137	1	132	4	96.4
Wheat	150	0	1	149	99.4

Additional studies are being conducted to determine if the multivariate Gaussian assumption on the distribution of features is justified, and if so, to improve the confidence with which the parameters of this distribution are learned. In the future more elaborate studies will be conducted, using the statistical classification technique with possible modification of the Gaussian assumption.

C. FEATURE SELECTION

a) Divergence Criterion

Parallel to the above work, a preliminary study has been undertaken to determine a procedure by which the frequency bands most important for the discrimination of agricultural crops can be found. It is hoped that these studies will aid in the reduction in dimensionality of the feature space.

Based on the multivariate Gaussian assumption on the distribution of features for each class, the divergence criterion of Marill and Green[1] was employed to determine the best subsets of features for recognition. This procedure can be used in the case of pair discrimination; i.e., for discrimination of two classes at a time. The results of this study were inconclusive in that it was determined that different subsets of features were important depending on the particular dichotomy being considered. These studies did lead to the following two conclusions:

1) Contrary to first thoughts, highly correlated features can be important for recognition. Based on the divergence criterion, this is true for the case in which the means of the two classes are widely separated in the correlated features.

2) Other methods for the selection of subsets of features should be considered.

Based on the second conclusion, another approach to the feature selection problem was undertaken.

b) Backward Programming

This approach to the feature selection problem is an outgrowth of work done in the construction of optimal stopping rules and optimum feature selection for sequential recognition processes. A brief description of the procedure is included here.

A sequential pattern recognition process can be described in the following way. We take a sequence of measurements of pattern features. After each measurement, a decision is made. This decision includes both the choice of closing the sequence of observations and classifying the pattern, or the choice of making another feature measurement before attempting classification. For situations in which the features are not identically distributed, we can seek to determine the best feature to measure next in light of which features remain to be measured, costs of respective feature measurements, costs of classification errors, and present knowledge. A solution to this double optimization problem can be obtained by a procedure based on backward programming, using the expected cost of the recognition process as the performance index. The procedure results in the calculation (at any stage of the sequential pattern recognition process) of the optimal decision rule for classification or feature selection for any sequence of feature measurements.

In addition, the backward programming procedure as implemented provides an optimum answer to the feature subset selection problem. That is, it can be used to determine the best subset of available features to use in a fixed size

Baysian or maximum likelihood pattern recognition procedure. This is done simply by setting the cost of measurements to zero. (This forces the classification decision to be made at the last stage of sequential pattern recognition process.) Then to determine the best subset of available features, all that need be done is to choose the feature subset of the desired size which minimizes the cost of the recognition process.

This procedure has the advantage that a Gaussian assumption is not employed, and situations involving more than two pattern classes can be handled. It has the disadvantage, as does the divergence criterion, that the number of computations required increases rapidly as larger sets of available features are considered.

Presently the backward programming procedure has been implemented to handle 3 pattern classes using 8 features. It is anticipated that it will be easier to increase the number of pattern classes than to increase the number of features. The program of this procedure has recently been checked out, and in the future the studies indicated above will be implemented using available LARS data.

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II. DATA HANDLING AND ANALYSIS

A. DATA HANDLING RESEARCH AND OPERATIONS

The data handling effort was established a) to make available to the data analysis researcher appropriate quantities of data in an easily used format, and b) to develop suitable equipment and techniques so that the data processing capability can grow at a rate commensurate with the rest of the remote sensing program. The major portion of time in the first months was spent generating specifications for the equipment required for data handling and analysis. In the last few months fragments of an important data handling programming system have developed into the first phase of an operational system. This data handling research and operations aspect of the remote sensing program is described below.

A digital approach was chosen to meet the data formating, editing, computation, and analysis needs of the project. This approach was deemed the most desirable because it provides flexibility, compatability, fidelity, speed, and evolutionary convenience and economy. Equipment procured for this purpose includes an Analog-to-Digital Conversion system, and an IBM System/360 Model 44 general purpose computer. Brief specifications of the A/D conversion system include:

1. Throughput of 100,000 eight bit samples per second
2. Maximum sampling of 140,000 eight bit samples per second
3. 16 multiplexed sample and hold channels
4. 7 track IBM compatable magnetic tape output
5. 16,384 six bit character core buffer between A/D converter and digital tape.

Features on the computer include:

1. 65,536 byte core storage

2. Floating point arithmetic
3. Three 60,000 bytes per second magnetic tape units
4. One 30,000 bytes per second magnetic tape unit
5. 600 line per minute printer
6. Card reader-punch.

The latter system has recently been received and placed in operation.

A third system, a digital display, is planned and specified for connection to a computer. Features of the display include:

1. Display of 500 x 500 16 level elements at 30 frames per second
2. Buffer memory for one frame
3. Light pen
4. A maximum of man-machine operation convenience.

IBM is now building a display with the above specifications.

Several data handling software systems or programs are required for the remote sensing program. These can be classified into two categories: Data Cataloging and Data Transformation. Large amounts of data coming from various sources, such as photographs, charts, tables, etc., are being cataloged on the computer to permit easy recall by the researcher for future analysis. The more difficult category is data transformation, which is the conversion of the raw data forms to a digital data form in a format easily used by the researcher. Data from the Block interferometer and aircraft scanner must be transformed before it is useful to the researcher.

The transformation process for data from the Block interferometers includes a digitizing process, an inverse Fourier transform process, and a formating and output process. Work has started on this system with a few samples of interferograms being digitized and inverse Fourier transformed. This system is expected to be operational for the spring growing season.

The more difficult data transformation problem involves the aircraft scanner data. The first phase system for scanner data includes a digitizing process, a calibration and reformating process, a display process, and an editing or selection process. The present data digitization is accomplished at the Marshall Space Flight Center, Huntsville, Alabama. Sixteen minutes of flight data were digitized onto 48 digital magnetic tapes. It should be noted that the same sixteen minutes will fit on 16 tapes when the LARS A/D system arrives.

The calibration and reformating process is a computer program which calibrates the digital data from calibration sources in the optical mechanical scanner. This is not an absolute calibration. The calibrated data is then reformated onto a data storage tape. At present this program is written for the IBM 7094, and is being rewritten for the IBM System/360 Model 44.

The current display process consists of a pictorial print-out on a line printer. One wavelength band or combination of wavelength bands is selected, and printed out in pictorial form with a simulated 16 level gray scale. From line numbers and column numbers printed on the pictorial print-out, the researcher can edit or select rectangular areas of interest. A program is written which will obtain the selected data from the data storage tape and output it in all wavelength bands in a format easily used by the researcher.

The display process and selection process can be grouped into one program when a display is available to the project. The TV-like picture will replace the pictorial print-out. Data can be selected by software interpretation of operator light pen or keyboard responses.

The present data handling systems are crude and require refinement. Due to the large amount of data to be handled to supply the research needs, the systems must be fast and automatic. The next phase of this work will include many improvements of current systems.

B. STATISTICAL ANALYSIS AND MODELING

To understand the variation of multispectral response patterns, the following statistical analyses of data are being studied, and some have been carried out with the available DK2 spectrophotometer data.

1. Assumption of a Model

A multispectral response pattern is assumed to be a sample from a multivariate normal distribution. Using this assumption, the parameters of the distributions (i.e., the mean vector and the covariance matrix) of each group are estimated from samples of that group, and multivariate statistical techniques can be applied to test hypotheses about the similarity or difference of multispectral response patterns from different groups.

2. Testing of Simple Hypotheses, and Analysis of Variance

i. Test the hypothesis that the variance covariance matrices of

a. different varieties of a given crop

b. a given crop under different conditions

c. different crops

are equal.

ii. Under the condition that the variance covariance matrices of different groups are equal, test the hypothesis that the mean vectors of different groups are equal.

3. Analysis of Principal Components

The method of principal components is used to find the linear combinations with significant variance. In many exploratory studies, the number of variables under consideration is too large. For example, the DK2 spectral data is sampled at every 10 $m\mu$ over the range of 0.5μ to 2.6μ. By considering the measurement at each sampled point as a variable, there will be 211 variables. Since it is the variations which are of interest, a way of reducing the number of variables is to discard the linear combinations which have small variances and study only those with large variances. The principal components give a new set of linearly combined measurements. It may be that three linear combinations will cover most of the variations from sample to sample. Preliminary analysis of the DK2 data seems to indicate that there are only three significant compounds.

4. Multiple Regression and Factor Analysis

The method of factor analysis is used to reduce a large number of correlated variables to terms of small uncorrelated variables which are hypothetical or measurable factors such as maturity of crop, soil type, moisture content.

C. FIELD SPECTROSCOPY IN AGRICULTURAL REMOTE SENSING

Field spectroscopy experiments were carried out in the 1966 growing season with a Perkin-Elmer SG-4 rapid scanning spectrometer. Corn, wheat, oats, alfalfa, soybeans, grasses, and a variety of soils were viewed in the spectral range from 0.35 to 4.07 microns. Viewing was done from either a pickup truck (close-up work) or a cherry-picker bucket (distance work and angle studies). Over 400 spectra were obtained from June through August.

Typical comparative spectra are shown in Figure 1 for the visible and near infrared region. The ability to distinguish all four subjects is manifest. In particular, the high reflectance of live vegetation in the near infrared stands out clearly. This high reflectance exists because light of these wavelengths is not absorbed, either by pigments or water, and a typical ray will undergo a random walk in the leaf microstructure until it reflects from or passes through

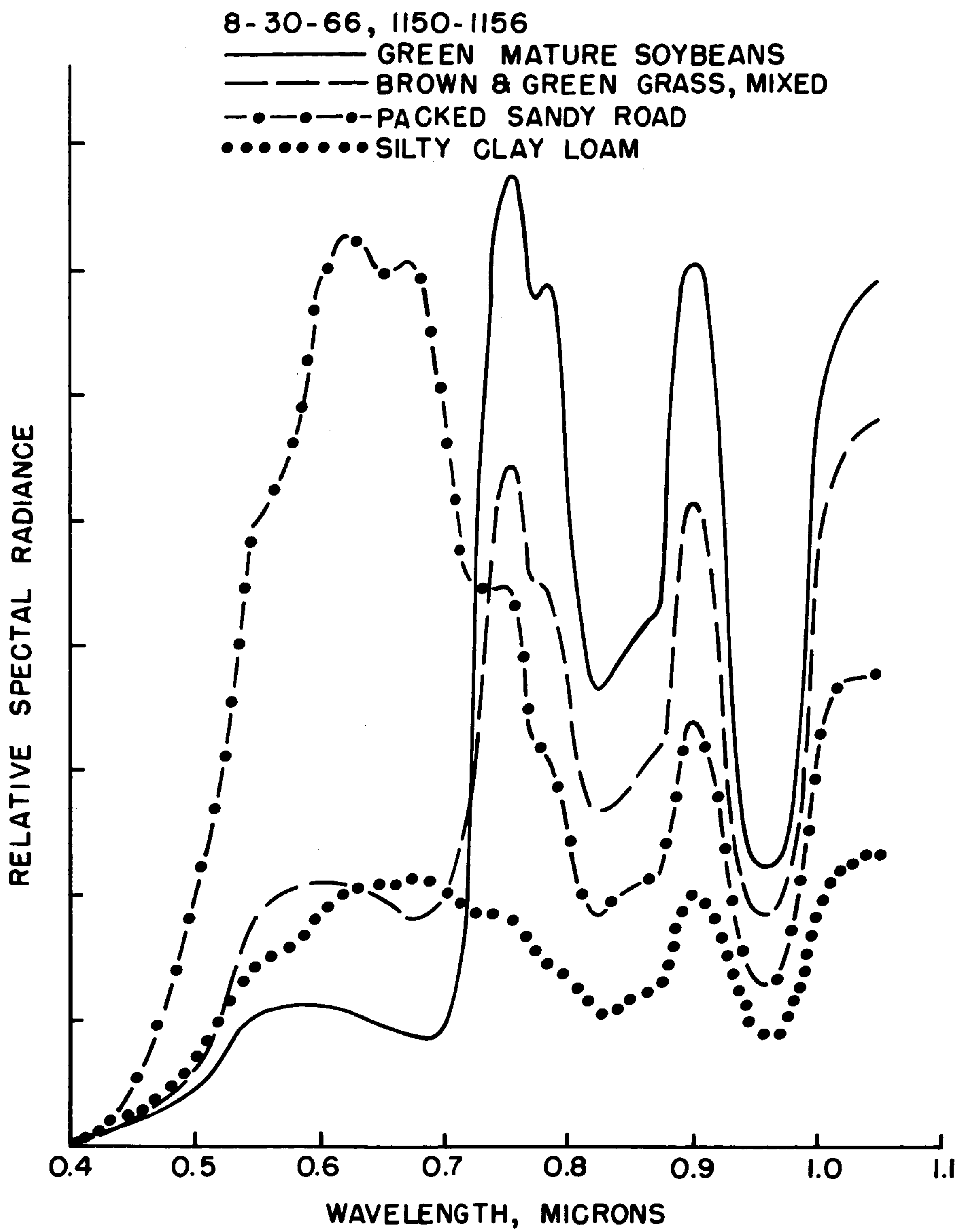

Fig. 1 Typical comparative spectra

the leaf.

Spectra of natural vegetation and soils do not show any pronounced detailed spectral structure differences beyond 1 micron. Instrument response in the atmospheric windows from 1 to 4 microns is similar in shape for a large variety of targets. The <u>total</u> reflected power in any of these atmospheric windows will be different for different targets. This indicates that high resolution spectroscopy in these windows is unnecessary.

Current work centers around the use of three Block interferometer spectrometers; these instruments produce Fourier transforms of the incident aperture irradiance. Spectral coverage is from 0.35 to 2.5 microns on a Soleil compensator interferometer, and from 2 to 6 and 2.5 to 16 microns on two Michelson interferometers, with very high sensitivity in the 2 to 6 micron instrument. These units will become part of a field van mobile spectroscopy facility now under construction. Two of the interferometers were used in the field in mid-September; the data reduction process was recently perfected, and spectra from these experiments will be available soon.

Winter and spring efforts will concentrate on calibration techniques in the reflective region out to 4 microns. Variable solar illuminance of earth due to cloud cover was a constant problem in 1966, and poses the major problem to be overcome in acquiring accurate reflectance neasurements in the field.

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